

Analytical and numerical approximations to the radiative transport equation for light propagation in tissues

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Abstract

In this lecture, we give a review of the assumptions used typically in applying transport theory to study light propagation in tissues. Then we give an overview of several analytical and numerical approximations to solve the radiative transport equation for direct problems that model measured data. In doing so, we identify several of the outstanding challenges in computing solutions for direct problems. Finally, we discuss these topics in the context of solving inverse problems of practical interest in the biomedical optical imaging community.

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