

## Optical tomography on simple Riemannian manifolds

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When the refractive index of a medium is constant, the dynamics of photons in a scattering and absorbing medium are modeled by the radiative transport equation. In the absence of interaction with the medium, photons follow straight lines. However, if the refractive index varies continuously then the trajectories are bent away from being straight. We model these dynamics by assuming a background Riemannian metric; photons which are not absorbed or scattered now follow the geodesics of this background metric.

We will present results for simple manifolds of all dimensions  $n \geq 2$ , but will focus on the more delicate and interesting problem in dimension two where the role of the geometry is more apparent. We show that knowledge of the albedo operator, that which maps incoming flux to outgoing flux at the boundary, uniquely determines the absorption and scattering properties of the medium. We will also discuss the question of stability for metrics close to the Euclidean metric, the situation relevant to practical implementation of optical tomography.

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