Helmholtz Inversion

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Abstract

WELL POSED INVERSION

Solution of the moment problem in an ill-posed and well-posed manner will be discussed, for purposes of motivating the inversion of PDE.

The inversion of the Helmholtz equation

$$grad(grad U(r)) + k^2[1 + f(r)]U(r), r \in V.$$

will be discussed, with V an open subset of $\mathbb{R}^3$. Inversion of this equation involves the reconstruction of $f(r)$ by insonifying V with "sources" U_I which satisfy the equation

$$grad(grad U_I(r)) + k^2 U^I(r) = 0, r \in V.$$

It will be shown that corresponding to any point r_0 in V, and given any positive number E, there are two sources which when "fired" in quick succession, and the response of each is measured at a point r on the boundary of V, enable the evaluation of $f(r_0)$ to within an error of E, after performing only one addition, one multiplication, and one division. This proposed procedure differs from algorithms in current use, which are iterative, and which require the computation of several "forward" solutions in V of the Helmholtz equation.

The approach illustrated in the talk can also be applied to the electric field integral equation, to the heat equation, and to the equation $grad.(sigma(r) grad U(r))) = 0 in V$.

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