Direct and inverse problems of the wave scattering by screens with arbitrary finite inhomogeneities

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Resume
We consider direct and inverse problems for a class of domains with noncompact boundaries that arise in mathematical models of the wave scattering by planar screens with arbitrary finite inhomogeneities. We prove the unique solvability of the direct problems in the Sobolev spaces. The inverse problems are formulated and uniqueness of reconstructing the permittivity and the shape of the scatterer from the scattering data is proved.

Abstract
In this study, we consider direct and inverse diffraction problems for screens with finite inhomogeneities. The screens form a class of cylindrical domains with noncompact boundaries and finite inhomogeneities that are simulated by a family (Π_R^∞) of unbounded two-dimensional cross-sectional domains Ω with noncompact (infinite) boundary contours. The geometry of these domains is such that their boundaries contain infinite rectilinear beams, so that they admit formulation of the boundary value problems for the Helmholtz equation with the Reichardt–Sveshnikov partial radiation conditions at infinity [1, 2], which is a generalization of the Sommerfeld conditions for the case of complex frequencies. Similar conditions [3] or their modifications [4, 5] were applied earlier (at real frequencies in [2] and at complex frequencies in [5]) to cylindrical domains (regular waveguides) with local inhomogeneities and to infinite domains with compact or noncompact periodic boundaries (see [4–6]). The results of [5, 7], where a class of domains similar to Π_R^∞ was introduced and boundary value problems with generalized partial radiation conditions were formulated, enable us to extend the settings and results to direct and inverse scattering problems in a half-plane or in a half-space with local inhomogeneities, which constitutes the essence of the present study.

The main results are published in [7, 16].

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    http://www.vsppub.com/jconts/JIP/jc-Vol11No6pp545654200.html