

Simple Riemannian metrics as minimal surfaces in Banach spaces

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Abstract

A Riemannian metric g on $D = D^n$ is said to be simple if every two points of D are connected by a unique minimal geodesic of g and no geodesic has conjugate points. It is conjectured that every simple metric is boundary rigid (i.e., uniquely determined by the boundary distance function) and is a minimal filling (i.e., its volume equals the filling volume of the boundary).

The conjectures are proved in a number of cases including $n = 2$ and the case when g is close to a flat metric. Some proofs are based on the following construction: (D, g) can be mapped isometrically into a Banach space (via a so-called Kuratowski embedding), and it turns out that the conjectures are equivalent to the following: the image of (D^n, g) is an absolutely minimal surface in that space.

I will explain this construction and show that a Kuratowski image of a simple metric is minimal in the variational sense, and minimizes the area locally in a certain topology.