Discretization invariant Bayesian inversion

Samuli Siltanen *

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Abstract

Bayesian inversion is considered for indirect measurement $M = AU + \mathcal{E}$ of a physical (continuum) quantity U. Here A is a smoothing linear operator and $\mathcal{E}$ is white noise. Practical data is a realization $\widehat{M}_k$ of the random variable $M_k = P_k AU + P_k \mathcal{E}$, where P_k is a linear orthogonal projection related to measurement device. To allow computerized inversion, the unknown is discretized as $U_n = T_n U$, leading to the computational measurement model $M_{kn} = P_k AU_n + P_k \mathcal{E}$. Bayes formula gives then the posterior distribution $\pi_{kn}(u_n \,|\, m_{kn}) \sim \Pi_n(u_n)\, \exp(-\frac{1}{2}\|m_{kn} - A_k u_n\|_2^2)$ in $\mathbb{R}^n$, and the mean $\mathbf{u}_{kn} := \int u_n\, \pi_{kn}(u_n \,|\, \widehat{M}_k)\, du_n$ is considered as the reconstruction of U. The choice of prior distributions Π_n for all $n \geq 1$ is called *discretization invariant* if the distribution of U_n converges to a limit distribution as $n \to \infty$, and the mean $\mathbf{u}_{kn}$ converges to a limit estimate as $k, n \to \infty$. General conditions assuring discretization invariance are proven using the novel concept of *reconstructor*, a deterministic map taking the measurement data almost surely to the mean. Gaussian smoothness priors and wavelet-based Besov space priors in the Besov space B^s_{pp} are shown to be discretization invariant. In particular, Bayesian inversion in dimension two with the edge-preserving B^1_{11} prior is related to using a prior penalizing the ℓ^1 norm of the wavelet coefficients of U.

*Tampere University of Technology