MILLIMAN LECTURES

János Kollár

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Each year the Mathematics Department invites a distinguished mathematician to visit the department for a week and deliver a series of lectures called the Milliman Lectures. This Lectureship is funded by the Milliman Fund, an endowed fund established in 1983 by a gift from Grace Milliman Pollack and her husband, S. Wilson Pollack, in honor of Mrs. Pollack's brother, W.A. Milliman, who received his Mathematics degree from the University of Washington in 1926 and who was a founder of the actuarial firm of Milliman and Robertson.

The 2002-03 Milliman Lecturer will be János Kollár of Princeton University, who is scheduled to visit the department during the week of April 14-18, 2003.

Kollár is generally considered to be one of the leading researchers in algebraic geometry today. He was one of three main organizers and one of the main speakers of the AMS Summer Research Institute on Algebraic Geometry in 1995, which is an event in algebraic geometry held once in every ten years, bringing together practically every algebro-geometer from around the world. It is a three-week long conference designed to cover the most important developments of the previous decade. This is the most important single event in algebraic geometry.

His first lecture, suitable for undergraduates, is titled "What is the biggest multiplicity of a root of a degree *d* polynomial?" The abstract is as follows:

> Let $f(x,y)$ be a real polynomial of degree d which has an isolated local minimum at the origin and $f(0,0)=0$. We say that it has a root of multiplicity M if $f(x,y)$ is $\leq c\sqrt{(x^2+y^2)^m}$ near the origin for some $c > 0$ and M is the smallest possible. We try to answer the question: how large can M be?

Kollár's second lecture, geared toward a colloquium level audience, is titled, "Which are the simplest algebraic varieties?" The abstract is as follows:

> An algebraic variety is a subset of C^n defined by polynomial equations. It is rather clear that the higher the degree of the defining equations, the more complicated the corresponding variety can be. There have been various approaches to define what the "most complicated" varieties are, but it is only recently that a good definition and theory was developed for the "simplest" varieties. The talk will explain the definition and its basic properties, mostly through examples.

Kollár was an invited speaker at the ICM'90 in Kyoto, where Shigefumi Mori, Kollár's main collaborator, received the Fields Medal. He was a plenary speaker at the European Congress of Mathematics in 1996.

His results cover such a wide area that it is hard to simply list them all. He has been an influential force in developing theories dealing with the Minimal Model Problem and the Iitaka conjecture regarding Kodaira dimension of fiber spaces, both of which are very important parts of classification theory. Along with Mori and Miyaoka he developed the theory of rationally connected varieties, which has made a tremendous impact on current research. His results on effective versions of the Hilbert Nullstellensatz make him one of the most quoted authors by people working on computational questions. He has achieved stunning advances in determining rationality, unirationality, ruledness and uniruledness of hypersurfaces that sparked a flurry of new research and new results by others. He obtained results on orbifolds using complex analytic techniques and on varieties in characteristic p using arithmetic methods. He has been one of the main architects of the theory of moduli spaces of higher dimensional varieties, proving many results, including projectivity of complete moduli spaces. He presented his work on Shafarevich's conjecture as the M. B. Porter Lecturer at Rice University. This was also published in a book that many collegues categorized as a very long research article reflecting the fact that the theory presented there was entirely new. In fact, his results on Shafarevich's conjecture are still the best (and basically the only) known results in the higher dimensional case.

He is the author or co-author of five books, some of which are regularly used for topics courses around the world.

In 1952 Nash proved that every compact differentiable manifold can be realized as the set of real points of a real algebraic manifold. He then went on to conjecture that every compact differentiable manifold can be realized as the set of real points of a real algebraic manifold, which is birational to projective space.

In 1998 Kollár proved that this conjecture fails in dimension 3 (and pointed out that it also fails in dimension 2 by work of Comessatti dated 1914). He gave a complete list of oriented real projective algebraic 3-folds that are birational to projective 3-space showing that in fact very few 3-manifolds can be realized this way.

This result is arguably the most significant single result in real algebraic geometry in recent years. Actually it is not the problem itself but Kollár's work on the problem that makes it so interesting.

Kollár has been known as one of the main architects of the Minimal Model Program (which was the main invention of the 1980's and constitutes the cornerstone of classification of complex algebraic 3-folds) and in the course of working on the Nash conjecture he developed the equivalent of the entire Minimal Model Program including some related "side results'" for real algebraic varieties.

This has not even been considered possible previously since real algebraic geometry seems to lie closer to differential geometry and topology than to algebraic geometry itself. The lion's share of algebro-geometric techniques require the field to be algebraically closed, so one would not think at first that such advanced theories as the Minimal Model Program would work for a non-closed field. That was before Kollár's work.

Naturally, his proof includes lots of topology and differential geometry, but it is also a very important link between algebraic geometry over the complex and the real numbers. Even though there is an obvious connection between the two, it is very hard to make good use of it in practice.