Sections 1.5 and 4.1 Review

In section 4.2 we will discuss the method of linear programming (a method we can use to find maximum and minimum values when we are selling/manufacturing two products). Before we can understand these methods, we need three skills:

- Solving systems of equations.
- Graphing regions given by inequalities.
- Translating a problem into a set of inequalities (we'll practice this in lecture when we discuss 4.2)

Solving Systems of Equations:
All methods involved **combining** the two equations. I never use the words 'set them equal'. In class, I talked about different methods (substitution and adding/subtracting equations), you are welcome to use any correct method, just show your work. In this review sheet, I will just use substitution, it always works and it requires the least amount of cleverness (*i.e.* no tricks needed, everyone can do it).
The method of *substitution* involves solving for one variable in one equation, then substitute into the other equation. Here are two examples:

1. Solve
$$\begin{array}{lll} \text{(i)} & -3x+2y & =4 \\ \text{(ii)} & 4x+6y & =38 \end{array}$$
(Note how I label the equations (i) and (ii), I think this is a good strategy to help you stay organized.)
   - Step 1: Solve for one variable from one equation:
   $$\text{(i)} \quad -3x+2y=4 \quad \Rightarrow \quad 2y=4+3x \Rightarrow \quad y=2+1.5x.$$
   - Step 2: Substitute into the other equation:
   $$\text{(i) and (ii)} \quad 4x+6(2+1.5x)=38 \quad \Rightarrow \quad 4x+12+9x=38 \quad \Rightarrow \quad 13x=26 \quad \Rightarrow \quad x=2$$
   And we can go back to either (i) or (ii) (or both) to find y. From (i): $y=2+1.5x=2+1.5(2)=5$. Thus, $(x,y)=(2,5)$ is the solution (and you can check your work, by seeing that this point satisfies both of the original equations).

2. Solve
$$\begin{array}{lll} \text{(i)} & 2x+5y & =7 \\ \text{(ii)} & 3x-2y & =2 \end{array}$$
   - Step 1: Solve for one variable from one equation:
   $$\text{(i)} \quad 2x+5y=7 \quad \Rightarrow \quad 5y=7-2x \Rightarrow \quad y=1.4-0.4x.$$
   - Step 2: Substitute into the other equation:
   $$\text{(i) and (ii)} \quad 3x-2(1.4-0.4x)=2 \quad \Rightarrow \quad 3x-2.8+0.8x=2 \quad \Rightarrow \quad 3.8x=4.8 \quad \Rightarrow \quad x=\frac{4.8}{3.8}\approx 1.26315789$$
   And we can go back to either (i) or (ii) (or both) to find y.
   From (i): $y=1.4-0.4x=1.4-0.4(1.26315789)=0.89473684$.
   Thus, $(x,y)\approx(1.263,0.895)$ is the solution (again you can check your work by seeing that this point satisfies both of the original questions).

Graphing Inequalities:
The key step in linear programming is graphing our inequalities. So most of your work in 4.1 and in 4.2 will involve the following skills. Here is a quick summary of the skills we discussed:

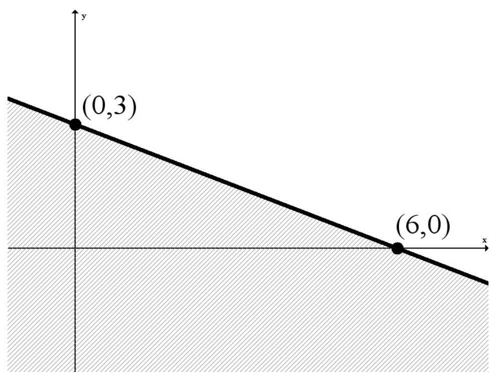
1. Given one inequality, graph the line and shade the appropriate side (you can use a sample point to figure out which side to shade).

2. Given multiple inequalities, graph each inequality separately, then shape in the overlapping region.

Here are three examples:

1. Graph $2x + 4y \leq 12$

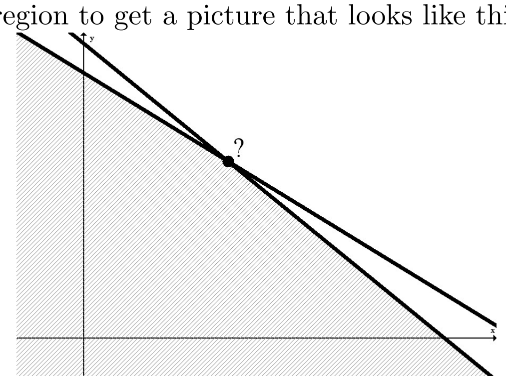
   - Step 1: Graph the line $2x + 4y = 12$. Find two points!
     When $x = 0$, we get $4y = 12$, so $y = 3$. Thus, $(0, 3)$ is a point on the line.
     When $y = 0$, we get $2x = 12$, so $x = 6$. Thus, $(6, 0)$ is a point on the line.
     Now draw this line.
   - Step 2: Take a 'sample point' from one side and put it into the equality to see if you shade that side. The origin $(0, 0)$ is on one side, let's use it as a sample point. In the inequality we get $2(0)+4(0) \leq 12$. Yes, this is true! So we shade the side that includes the point (0,0). Here is the region given by this inequality:



2. Graph $x + y \leq 100$ and $3x + 4y \leq 360$ and shade the overlapping region.
   Do each line separately (like we did in the last example)

   - $x + y = 100$ goes through $(0, 100)$ and $(100, 0)$ and shade the origin side.
   - $3x + 4y = 360$ goes through $(0, 90)$ and $(120, 0)$ and shade the origin side.

   Then shade the overlapping region to get a picture that looks like this:



   Aside: You should know how to find the point I marked with a question mark. It is the intersection point of $x + y = 100$ and $3x + 4y = 360$ (use the methods from the previous page and you should get $x = 40$ and $y = 60$).

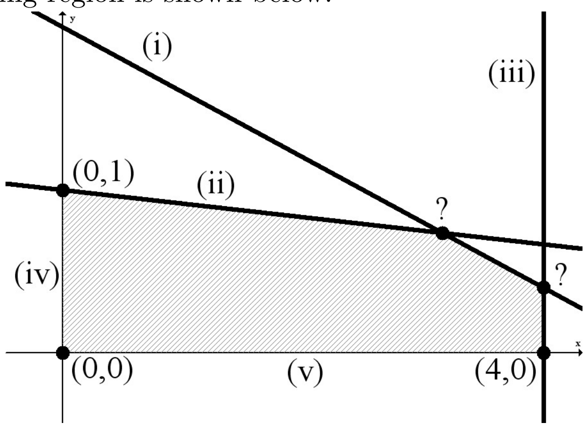
3. Graph the overlapping region given by:

$$\begin{array}{lrl} \text{(i)} & 2x+5y & \leq 10 \\ \text{(ii)} & x+12y & \leq 12 \\ \text{(iii)} & x & \leq 4 \\ \text{(iv)} & x & \geq 0 \\ \text{(v)} & y & \geq 0 \end{array}$$

(a) Step 1: Do each lines separately (like we did in the last example)

- $2x + 5y = 10$ goes through $(0, 2)$ and $(5, 0)$ and shade the origin side.
- $x + 12y = 12$ goes through $(0, 1)$ and $(12, 0)$ and shade the origin side.
- $x = 4$ is a vertical line at $x = 4$ and shade the origin side.
- $x = 0$ is the y-axis and shade to the right of it.
- $y = 0$ is the x-axis and shade above it.

(b) Step 2: The overlapping region is shown below:



Note: In order to do the application problems in 4.2, we need to find the corners of this region. Two of these corners are unknown so far (I have marked them in the picture). To find these, we need to do some intersecting. One is the point of intersection of (i) $2x + 5y = 10$ and (ii) $x + 12y = 12$ and the other is the intersection of (i) $2x + 5y = 10$ and (iii) $x = 4$.

- Intersecting (i) and (ii): Solving for y in (i) gives $5y = 10 - 2x \Rightarrow y = 2 - 0.4x$.
  Combining (i) and (ii) gives

  $$x + 12(2 - 0.4x) = 12 \quad \Rightarrow \quad x + 24 - 4.8x = 12 \quad \Rightarrow \quad -3.8x = -12 \quad \Rightarrow \quad x = \frac{12}{3.8} \approx 3.1578947$$

  And from this we get $y = 2 - 0.4x \approx 2 - 0.4(3.1578947) \approx 0.73684211$.
  So the intersection of (i) and (ii) is at $(x, y) \approx (3.158, 0.737)$.

- Intersecting (i) and (iii):

  $$\text{Combining (i) and (iii):} \quad 2(4) + 5y = 10 \quad \Rightarrow \quad 5y = 2 \quad \Rightarrow \quad y = \frac{2}{5} = 0.4$$

  So the intersection of (i) and (iii) is at $(x, y) \approx (4, 0.4)$.