Sections 1.6 Review

Business Functions with Algebra: The first part of this section reviews the business terms, but uses linear functions as opposed to graphs.

1. When a problem says "Profit is linear" or "Total Revenue and Costs are linear", then you know that their functions will take the form $f(x) = mx + b$. (See the previous review sheets for advice about finding the equation of a line).

2. If the market price is a **constant** p dollars/item, then $TR(x) = px$ and $\overline{MR}(x) = p$.
   For example, if each item sells for \$3.50, then $TR(x) = 3x$ and $\overline{MR}(x) = 3$.
   (Recall, graphically TR is a diagonal line with slope 3 and MR is a horizontal line at 3).

3. If the $FC = c$ and the average variable cost per item is a **constant** m dollars/item, then $TC(x) = mx + c$.
   For example, if fixed costs are \$1552 and average variable costs are a constant \$7 per unit,
   then $TC(x) = 7x + 1552$ and $\overline{MC}(x) = 7$.
   (Graphically TC is a line with y-intercept at 1552 and slope of 7, and MC is a horizontal line).

4. If given TR and TC and asked for the 'quantity at which you break even', then you need to solve $TR(x) = TC(x)$ (this is NOT asking about 'break even price', it is asking when profit would be zero for these particular TR and TC functions).

5. If you have the functions for $TR(x)$ and $TC(x)$, then you can get the functions for profit, average cost, average variable cost and average revenue by using the definitions. Just make sure that when you use parantheses and properly distribute when you substitute the functions into the definitions.
   Here is an example: Suppose $TR(x) = 5x$ and $TC(x) = 3x + 10$.

   - Profit $= P(x) = TR(x) - TC(x) = (5x) - (3x + 10) = 5x - 3x - 10 = 2x - 10$
   - Marginal Cost $= \overline{MC}(x) = 3$ (cost to produce next item)
   - Average Cost $= AC(x) = \frac{TC(x)}{x} = \frac{(3x+10)}{x} = 3 + \frac{10}{x}$ (average overall cost per item, including FC)
   - Average Variable Cost $= AVC(x) = \frac{(3x)}{x} = 3$ (average cost per item, excluding FC).
   - Marginal Revenue $= \overline{MR}(x) = 5$ (revenue from next item)
   - Average Revenue $= AR(x) = \frac{(5x)}{x} = 5$ (average overall revenue per item)

   You might have noticed that $MR(x) = AR(x) = 5$ and $MC(x) = AVC(x) = 3$ in this example. That will only happen when working with linear TR and TC functions (the slope of the diagonal is the same as the slope of the tangent in this case). When we work with more complicated functions this won't be the case.

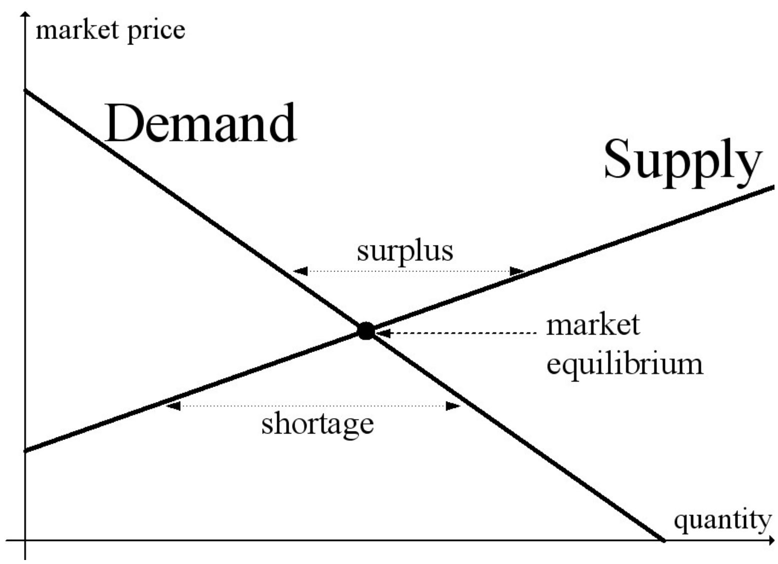
Supply and Demand: We introduced the concepts of supply and demand and we did examples with linear functions.

- A **supply curve** shows the relationship between market price, p, and the quantity, q, of a product the manufacturers are willing to supply for that market price.
  For example, if $(q, p) = (45, 300)$ is a point on the supply curve, then that means that at a market price of 300 dollars/item, the manufactures are willing to produce 45 units.

- A **demand curve** shows the relationship between market price, p, and the quantity, q, of a product that consumers will purchase at that price.
  For example, if $(q, p) = (70, 155)$ is a point on the demand curve, then that means that at a market price of 155 dollars/item, consumers will purchase 70 units.

- Some basic properties:
  - (The Law of Supply): The number of quantities supplied will increase as the market price goes up. (In other words, when you draw the supply curve it will go up as you go from left to right).
  - (The Law of Demand): The number of quantities demanded will decrease as the market price goes up. (in other words, when you draw the demand curve it will go down as you go from left to right).

- The quantity and price at which supply and demand intersect is called **market equilibrium**. This gives the price at which the manufacturers and consumers are willing to produce and buy the same number of units.
  - If the market price is greater than market equilibrium price, then there will be a surplus (more items will be produced than sold).
  - If the market price is less than market equilibrium price, then there will be a shortage (more items will be demanded than are produced).



- We did examples where two data points where given for Supply and two data points were given for Demand. Then we found linear functions for Supply and Demand (again, see the 1.1-1.3 for review of how to find equations for lines). Finally we found their intersection to find the market equilibrium.

- Here is a quick example: Suppose supply and demand are linear. You are told that at a price of \$150, the demand will be 50 units and the supply will be 10 units, and at a price of \$300, the demand will be 30 units and the supply with be 60 units. Find market equilibrium.
  Answer on the next page (try the problem before you look at the answer).

Answer to example from previous page:

1. Demand: The given (q, p) points are $(50, 150)$ and $(30, 300)$. Slope $= m = \frac{150-300}{50-30} = \frac{-150}{20} = -7.50$. So the equation for the line is

$$p = -7.50(q - 30) + 300 = -7.5q + 225 + 300 = -7.5q + 525$$

2. Supply: The given (q, p) points are $(10, 150)$ and $(60, 300)$. Slope $= m = \frac{150-300}{10-60} = \frac{-150}{-50} = 3$. So the equation for the line is

$$p = 3(q - 10) + 150 = 3q - 30 + 150 = 3q + 120$$

3. Market equilibrium: We have the same price when

$$-7.5q + 525 = 3q + 120$$

$$405 = 10.5q$$

$$q = \frac{405}{10.5} \approx 38.5724$$

which gives

$$p \approx 3(41.4286) + 120 \approx 235.714.$$

Rounding to the nearest item and nearest cent (this would depend on units of the problem), market equilibrium is at 39 units and a price of \$235.71.