Sections 2.1 Review

We introduce quadratic functions in this section.

Basic Quadratic Facts:

1. A **quadratic equation** is any equation that can be written in the form $ax^2 + bx + c = 0$, where a, b and c are given numbers. (It's important to note that one side is zero and the only variable is x).

2. To solve a quadratic equation:

   (a) Simplify/Clear Denominator: Always clear the denominators and look for easy simplifications (sometimes you can solve by just simplifying).

   (b) Make one side zero: Add/Subtract to get all terms to one side.

   (c) Factor/Quadratic Formula: If you can factor, do it. Otherwise, use the quadratic formula.

   (d) Check your answer

   Here is the quadratic formula:

   $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

   Note that

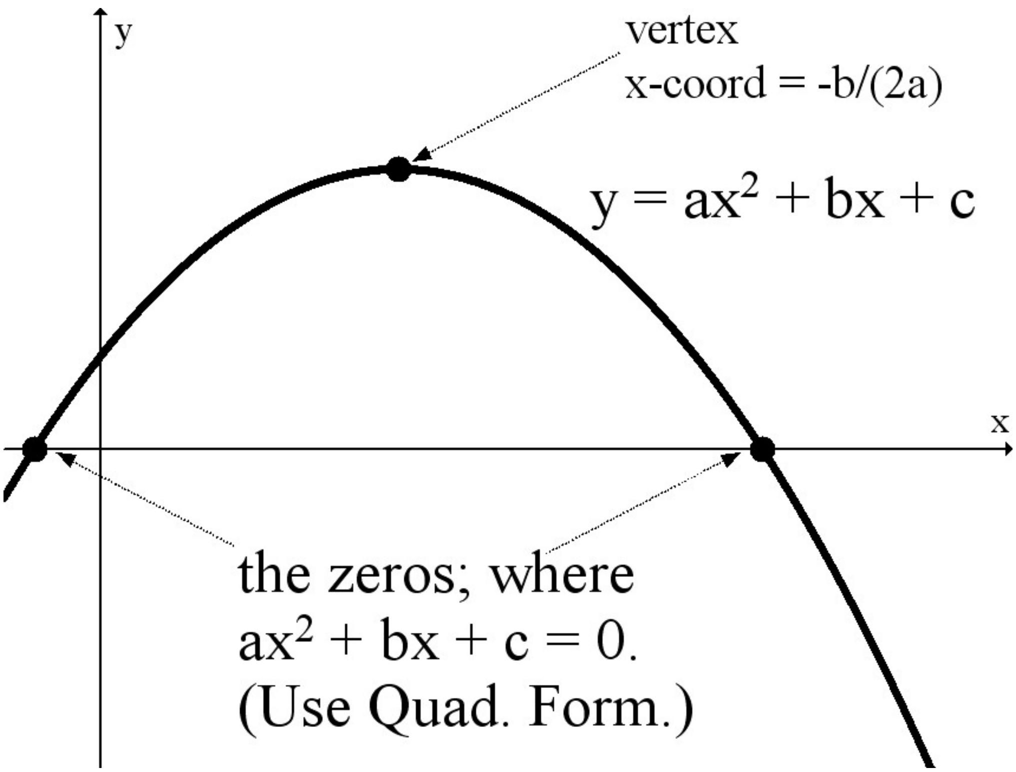
   - if $b^2 - 4ac$ is positive, then there are two solutions.
   - if $b^2 - 4ac$ is zero, then there is one solution.
   - if $b^2 - 4ac$ is negative, then there are no solutions.

3. A **quadratic function** is any function that can be written in the form $f(x) = ax^2 + bx + c$, where a, b, and c are given numbers. (Note that $f(x)$ is the name of the output, so there are two variable here, x is the input and $y = f(x)$ is the output).

4. The graph of the quadratic function $y = ax^2 + bx + c$ is called a parabola. The point where the parabola changes direction is called the vertex. Here are three important facts:

   - If $a > 0$, then the parabola opens upward (it smiles).
   - If $a < 0$, then the parabola opens downward (it frowns).
   - The x-coordinate of the vertex is located at $x = -\frac{b}{(2a)}$.

A picture of a quadratic function, $f(x) = ax^2 + bx + c$. Note that a must be negative (the parabola opens downward).



For extra practice with solving quadratic equations and using the quadatic formula, try these (solutions are online in the full review sheet).

1. $\frac{x^2}{3} + 1 = 5$
2. $(x-1)^2 + 4 = 13$
3. $(3x)^2 - 4 = 10.$
4. $x^2 - 6x + 5 = 0$
5. $x^2 - 7x + 1 = 3 - 2x$
6. $x^2 + 5x + 12 = -10 - 2x^2$

For practice finding the vertex, try these (solutions are online in the full review).
Find the x and y coordinates of the vertex for the following quadratic functions:

1. $y = 3x^2 - 60x + 5.$
2. $y = 46x - 2x^2.$
3. $f(x) = (x^2 + 4x) - (10x - 7).$

Answers to Solving Quadratic Equation Problems:

1. Answer:

$\frac{x^2}{3}+1 = 5$	
$\frac{x^2}{3} = 4$	subtracted 1 from both sides
$x^2 = 12$	multiplied both sides by 3
$x = \pm\sqrt{12}$	took the square root of both sides (and wrote both answers)

The two answers are $x = \sqrt{12}$ and $x = -\sqrt{12}$.

2. Answer:

$(x-1)^2+4 = 13$	
$(x-1)^2 = 9$	subtracted 4 from both sides
$x-1 = \pm\sqrt{9} = 3$	took the square root of both sides (and wrote both answers)
$x = 1 \pm 3$	added 1 to both sides

The two answers are $x = 1+3 = 4$ and $x = 1-3 = -2$.
Note: You can also expand $(x-1)^2 = x^2-2x+1$ at the beginning of the problem and use the quadratic formula. That will give the correct answers (it just will take a bit more work in my opinion).

3. Answer:

$(3x)^2-4 = 10$	note that $(3x)^2 = (3x)(3x) = 9x^2$
$9x^2 = 14$	simplified and added 4 to both sides
$x^2 = \frac{14}{9}$	multiplied both sides by 9
$x = \pm\sqrt{\frac{14}{9}}$	took the square root of both sides (and wrote both answers)

The two answers are $x = \sqrt{\frac{14}{9}} = \frac{\sqrt{14}}{3}$ and $x = -\sqrt{\frac{14}{9}} = -\frac{\sqrt{14}}{3}$. (You don't have to rewrite your answer in the simplest form as I did)

4. Answer:

Method 1: Factoring

$x^2-6x+5 = 0$	
$(x-1)(x-5) = 0$	factored (I noticed that $(-1)(-5) = 5$ and $(-1)+(-5) = -6$)

Thus, we can conclude $x-1 = 0$ or $x-5 = 0$, which gives $x = 1$ or $x = 5$.

Method 2: Quadratic Formula

$x^2-6x+5 = 0$	one side is zero and $a = 1$, $b = -6$, $c = 5$.
$x = \frac{-(-6)\pm\sqrt{(-6)^2-4(1)(5)}}{(2(1))}$	quadratic formula
$x = \frac{6\pm\sqrt{36-20}}{2}$	simplifying
$x = \frac{6\pm\sqrt{16}}{2}$	simplifying
$x = \frac{6\pm4}{2}$	simplifying

Thus, we conclude $x = \frac{6-4}{2} = 1$ or $x = \frac{6+4}{2} = 5$.

5. Answer:

$x^2-7x+1 = 3-2x$	make one side zero!
$x^2-5x-2 = 0$	subtracted 2 and added $2x$ to both sides
$x = \frac{-(-5)\pm\sqrt{(-5)^2-4(1)(-2)}}{(2(1))}$	quad. form. $a = 1$, $b = -5$, $c = -2$
$x = \frac{5\pm\sqrt{25+8}}{2}$	simplifying
$x = \frac{5\pm\sqrt{33}}{2}$	simplifying

Thus, we conclude $x = \frac{5-\sqrt{33}}{2}$ or $x = \frac{5+\sqrt{33}}{2}$.

6. Answer:

$x^2 + 5x + 12$	$= -10 - 2x^2$	make one side zero!
$3x^2 + 5x + 22$	$= 0$	added 10 and added $2x^2$ to both sides
x	$= \frac{-(5) \pm \sqrt{(5)^2 - 4(3)(22)}}{(2(3))}$	quad. form. $a = 3$, $b = 5$, $c = 22$
x	$= \frac{-5 \pm \sqrt{25 - 264}}{6}$	simplifying
x	$= \frac{5 \pm \sqrt{-239}}{6}$	negative under radical!!!

Since there is a negative under the radical, we conclude there is no solution.

Note: In the problems where I used the quadratic formula, see how I broke up my use of the formula into steps. (1) Simplify the expression under the radical, (2) simplify the denominator, (3) take the square root, then (4) simplify your answer. You should do the same and you should write out your work on paper. It is very common to make errors when you try to type it all into your calculator at once, so work it in steps and write down your intermediate work!

Answers to Vertex of a Quadratic Function Problems:

1. $y = 3x^2 - 60x + 5$.
   The x-coordinate of the vertex is $x = -\frac{(-60)}{2 \cdot 3} = \frac{60}{6} = 10$.
   The y-coordiante of the vertex is $y = 3(10)^2 - 60(10) + 5 = 300 - 600 + 5 = -295$.

2. $y = 46x - 2x^2 = -2x^2 + 46x$.
   The x-coordinate of the vertex is $x = -\frac{46}{2 \cdot (-2)} = \frac{46}{4} = 11.5$.
   The y-coordiante of the vertex is $y = 46(11.5) - 2(11.5)^2 = 529 - 264.5 = 264.5$.

3. $f(x) = (x^2 + 4x) - (10x - 7) = x^2 - 6x + 7$.
   The x-coordinate of the vertex is $x = -\frac{(-6)}{2 \cdot (1)} = 3$.
   The y-coordiante of the vertex is $f(3) = (3)^2 - 6(3) + 7 = 9 - 18 + 7 = -2$.