Sections 4.2 Review

The method of **linear programming** is a procedure we will use to optimize (max/min) an *objective* subject to *constraints*. Here is the short version:

- **Step 1: Label the two quantities.**
  Example: "How many pounds of each ... " would mean x = amount of first (in pounds), y = amount of second (in pounds)
- **Step 2: Collect information from the problem.** Make a table of rates for x and y, write out formulas for total amounts, and list any restrictions.
- **Step 3: Give Constraints and Objective.** Write down the inequalities for the constraints and write down the function for the objective. Note that the last sentence typically tells you the objective. For example, "...to minimize cost?" means that the cost function, $C(x, y)$, is the objective you are minimizing. You will only use the objective at the end of the problem.
- **Step 4: Sketch the Feasible Region.** Draw the line and shade the region that corresponds to each of the constraint inequalities. Indicate the overlapping region (this is the feasible region).
- **Step 5: Find all the corners for the overlapping Feasible Region.** You probably will need to do some intersections to do this. Use your picture to figure out which lines you need to intersect.
- **Step 6: Plug each corner into the objective.** The largest output is the maximum and the smallest output is the minimum.

We did four big examples in lecture. Here are a three more for you to try (full answers are on the following pages):

Lawn Care Example: A lawn care company has two types of fertilizer Regular and Deluxe. The profit for Regular is \$0.75 per bag and the profit for Deluxe is \$1.20 per bag. One bag of Regular has 3 pounds of active ingredient and 7 pounds of inert ingredients. One bag of Deluxe has 4 pounds of active ingredients and 6 pounds of inert ingredients. The warehouse has a limit of 8400 pounds of active ingredient and 14100 pounds on inert ingredient.
How many bags of each should we make to maximize profit?

A Boring No Words Question: Find the maximum of the objective $f(x, y) = 14x + 20y$ subject to the constraints: $4x + 6y \leq 1800$, $x \leq 300$, $y \leq 150$, $x \geq 0$, and $y \geq 0$.

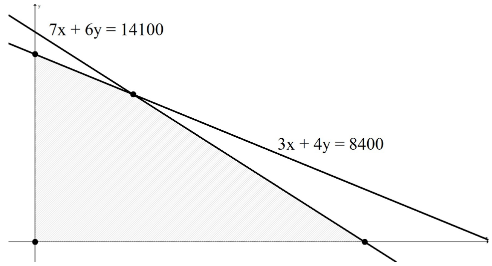
Cookie Company Example: Your company makes two types of chocolate chip cookies. Each bag of 'Chocolate Lite' contains 10 ounces of chocolate chips and 35 ounces of dough. Each bag of 'Chocolate Overload' contains 20 ounces of chocolate chips and 25 ounces of dough. The profit on each bag of Chocolate Lite is \$1.10, while the profit on each bag of Chocolate Overload is \$0.80. Your company current has 1000 ounces of chocolate chips and 2555 ounces of cookie dough.
How many bags of each should you produce to maximize profit?

ANSWER to Lawn Care Example:

Let x = number of bags of Regular and y = number of bags of Deluxe. Here is the collected information:

	x	y	Total formula	Constraints/Objective
Active	3	4	$3x + 4y$	$3x + 4y \leq 8400$
Inert	7	6	$7x + 6y$	$7x + 6y \leq 14100$
Profit	0.75	1.20	0.75x+1.20y	$P(x, y) = 0.75x + 1.20y =$ Objective

Now we get points, draw the lines and shade the feasible region:
$3x + 4y = 8400$ goes through $(0, \frac{8400}{4}) = (0, 2100)$ and $(\frac{8000}{3}, 0) = (2800, 0)$.
$7x + 6y = 14100$ goes through $(0, \frac{14100}{6}) = (0, 2330)$ and $(\frac{141000}{7}, 0) \approx (2014.29, 0)$.
Note that $(0, 0)$ works in both of these constraints so we shade the overlapping 'origin-side' of the lines.



To get the corner at the intersection, we combine: (i) $3x + 4y = 8400$ and (ii) $7x + 6y = 14100$
(i) $3x + 4y = 8400 \quad \Rightarrow \quad 4y = 8400 - 3x \quad \Rightarrow \quad y = 2100 - 0.75x.$
(i) and (ii) $7x + 6(2100 - 0.75x) = 14100 \quad \Rightarrow \quad 7x + 12600 - 4.5x = 14100 \quad \Rightarrow \quad 2.5x = 1500 \quad \Rightarrow \quad x = \frac{1500}{2.5} \approx 600.$
And $y = 2100 - 0.75x = 2100 - 0.75(600) = 1650$

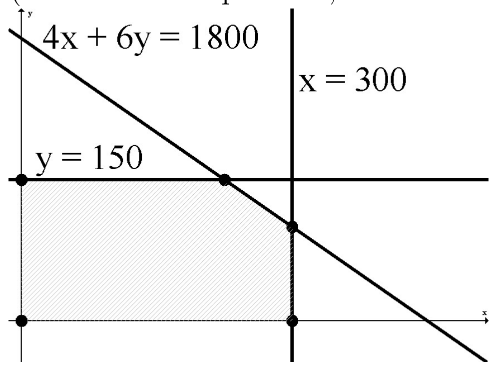
Evaluating the objective at the found corners gives:

$$\begin{aligned}
P(0,0) &= 0.75(0) + 1.20(0) &&= 0 \text{ dollars} \\
P(0,2100) &= 0.75(0) + 1.20(2100) &&= 2520 \text{ dollars} \\
P(2014.29,0) &= 0.75(2014.29) + 1.20(0) &&= 1510.71 \text{ dollars} \\
P(600,1650) &= 0.75(600) + 1.20(1650) &&= 2430 \text{ dollars}
\end{aligned}$$

Thus, the maximum profit is \$2520 and it occurs when you produce $x = 0$ bags of Regular and $y = 2100$ bags of Deluxe.

ANSWER to A Boring No Words Question:

The problem is already set up for us, so we just need to graph and find corners. First get points, draw the lines and shade the feasible region:
$4x + 6y = 1800$ goes through $(0, \frac{1800}{6}) = (0, 300)$ and $(\frac{1800}{4}, 0) = (450, 0)$.
$y = 150$ is a horizontal lines at $y = 150$.
$x = 300$ is a vertical line at $x = 300$. Note that $(0, 0)$ works in these three constraints so we shade the overlapping 'origin-side' of the lines (inside the first quadrant, that is what $x \geq 0$ and $y \geq 0$ is telling us).



To get the corners at the intersections, we combine.
One corner: (i) $4x + 6y = 1800$ and (ii) $y = 150$
(i) and (ii) $4x + 6(150) = 1800 \Rightarrow 4x + 900 = 1800 \Rightarrow 4x = 900 \Rightarrow x = \frac{900}{4} \approx 225$. So one of the corners is $(225, 150)$.
The other corner: (i) $4x + 6y = 1800$ and (ii) $x = 300$
(i) and (ii) $4(300) + 6y = 1800 \Rightarrow 1200 + 6y = 1800 \Rightarrow 6y = 600 \Rightarrow y = \frac{600}{6} \approx 100$. So the other corner is $(300, 100)$.

Evaluating the objective at the found corners gives:

$$\begin{aligned}
f(0,0) &= 14(0) + 20(0) &&= 0 \\
f(0,150) &= 14(0) + 20(150) &&= 3000 \\
f(300,0) &= 14(300) + 20(0) &&= 4200 \\
f(300,100) &= 14(300) + 20(100) &&= 6200 \\
f(225,150) &= 14(225) + 20(150) &&= 6150
\end{aligned}$$

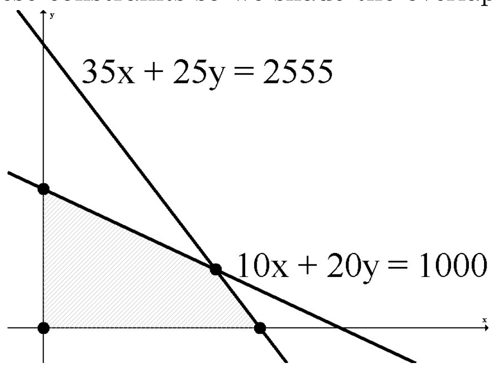
Thus, the maximum is 6200 and it occurs when $x = 300$ and $y = 100$.

ANSWER to Cookie Company Example:

Let x = number of bags of Chocolate Lite and y = number of bags of Chocolate Overload. Here is the collected information:

	x	y	Total formula	Constraints/Objective
Chips	10	20	$10x + 20y$	$10x + 20y \le 1000$
Dough	35	25	$35x + 25y$	$35x + 25y \le 2555$
Profit	1.10	0.80	1.10x+0.80y	$P(x, y) = 1.10x + 0.80y$ = Objective

Now we get points, draw the lines and shade the feasible region:
$10x + 20y = 1000$ goes through $(0, \frac{1000}{20}) = (0, 50)$ and $(\frac{1000}{10}, 0) = (100, 0)$.
$35x + 25y = 2555$ goes through $(0, \frac{2555}{25}) = (0, 102.2)$ and $(\frac{2555}{35}, 0) \approx (73, 0)$.
Note that $(0, 0)$ works in both of these constraints so we shade the overlapping 'origin-side' of the lines.



To get the corner at the intersection, we combine: (i) $10x + 20y = 1000$ and (ii) $35x + 25y = 2555$
(i) $10x + 20y = 1000 \Rightarrow 20y = 1000 - 10x \Rightarrow y = 50 - 0.5x$.
(i) and (ii) $35x + 25(50 - 0.5x) = 2555 \Rightarrow 35x + 1250 - 12.5x = 2555 \Rightarrow 22.5x = 1305 \Rightarrow x = \frac{1305}{22.5} \approx 58$.
And $y = 50 - 0.5x = 50 - 0.5(58) = 21$.

Evaluating the objective at the found corners gives:

$$
\begin{aligned}
P(0,0) &= 1.10(0) + 0.80(0) &&= 0 \text{ dollars} \\
P(0,50) &= 1.10(0) + 0.80(50) &&= 40 \text{ dollars} \\
P(73,0) &= 1.10(73) + 0.80(0) &&= 80.30 \text{ dollars} \\
P(58,21) &= 1.10(58) + 0.80(21) &&= 80.60 \text{ dollars}
\end{aligned}
$$

Thus, the maximum profit is \$80.60 and it occurs when you produce $x = 58$ bags of Chocolate Lite and $y = 21$ bags of Chocolate Overload.