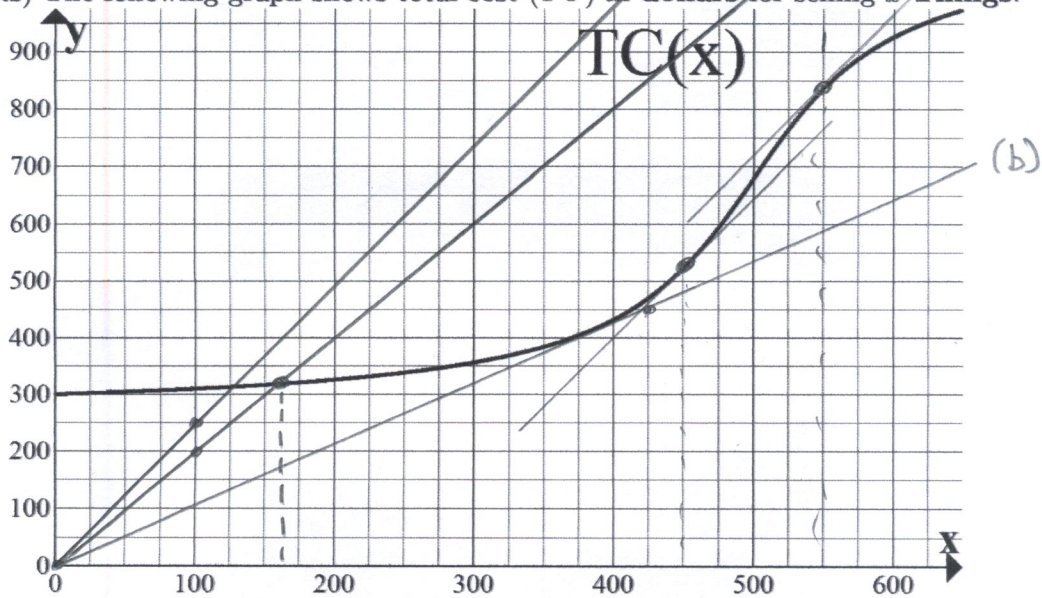


1. (14 points) The following graph shows total cost (TC) in dollars for selling x Things.



For each part, clearly explain your work in a sentence and label your work in the graph.

(a) Estimate the following values using the graph and the definitions:

- i. The fixed costs: $FC = \underline{300}$ dollars
- ii. The variable cost at $x = 450$: $VC(450) = \underline{\approx 225}$ dollars
- iii. The average variable cost at $x = 450$: $AVC(450) = \underline{\approx 0.50}$ dollars/Thing

(b) Find the breakeven price (BEP).

2 PTS: $(0,0)$ $(425, 450)$
 $SLOPE = \frac{450 - 0}{425 - 0} \approx 1.0588$

$BEP = \underline{\approx 1.06}$ dollars per Thing

(c) If the market selling price is below BEP, what does that tell you? (On the lines provided write one brief sentence/phrase answering this question)

It is NOT possible to make a positive profit
 if price is less than BEP

(d) Find the range of quantities over which marginal cost is at least 2.50 dollars per Thing.

$(100, 250)$

$x = \underline{\approx 450}$ to $x = \underline{\approx 550}$ Things

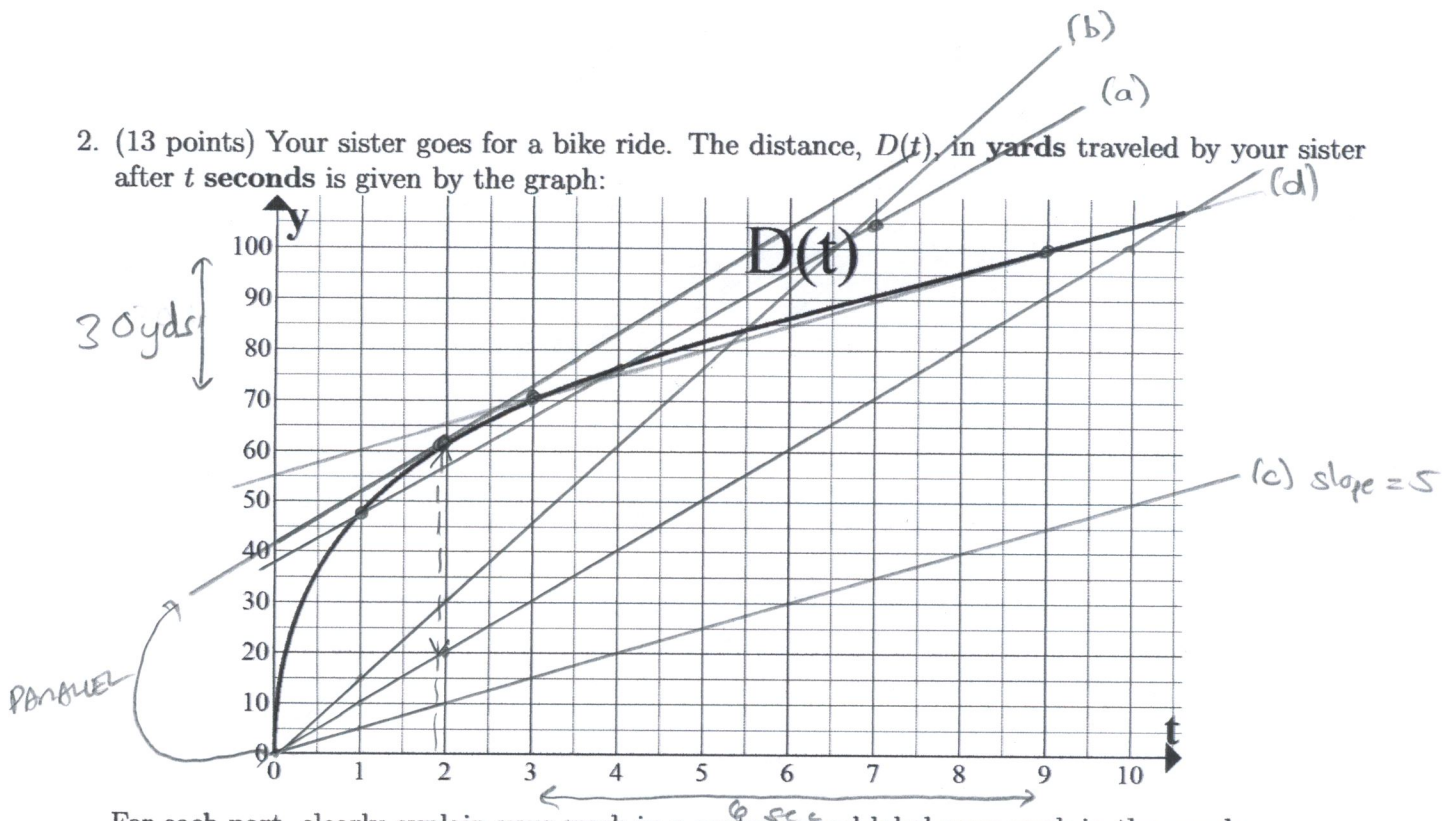
(e) Suppose the market selling price is fixed at 2 dollars per Thing.

Find the quantity at which profit is zero.

$(100, 200)$

$x = \underline{\approx 160}$ Things

2. (13 points) Your sister goes for a bike ride. The distance, $D(t)$, in yards traveled by your sister after t seconds is given by the graph:



For each part, clearly explain your work in a sentence and label your work in the graph.

(a) Find the average speed over the 3-second interval starting at $t = 1$ second. (Give the units)

2 PTS: $(7, 105)$
 $(1, 38)$

$$\frac{105 - 38}{7 - 1} \approx 9.57$$

Average Speed = $\boxed{\sim 9.57}$ UNITS = $\frac{\text{yards}}{\text{sec}}$

(b) Find a time when the average trip speed is 15 yards/second.

$(1, 15)$ $(2, 30)$

$t = \boxed{\sim 5.5}$ seconds

(c) Find a 6-second interval over which your sister travels 30 yards.

$\leftarrow 6 \text{ sec} \rightarrow$ $\uparrow 30 \text{ yards} \Rightarrow \text{slope} = 5$

$t = \boxed{\sim 3}$ to $t = \boxed{\sim 9}$ seconds

(d) You decide to go for a bike ride as well. You start at the same time and place, but you travel at a constant speed of 10 yards/second. Find the time when your sister is farthest ahead of you and estimate the distance between you at this time.

$(1, 10)$, $(2, 20)$

$\approx 61 - 20 = 41$

MATCH SLOPES

$t = \boxed{\sim 1.9}$ seconds
Distance ahead = $\boxed{\sim 41}$ yards

3. (11 points) You sell Things.

The total cost for selling x Things is $TC(x) = 14x + 2000$ dollars.

The selling price per Thing is $p = -12x + 600$ dollars/Thing.

(a) Give the formulas for total revenue, $TR(x)$.

$$TR = (\text{PRICE})(\text{QUANTITY})$$

$$TR(x) = -12x^2 + 600x$$

(b) Compute the marginal revenue and marginal cost at $x = 3$ Things.

$$MC(3) = TC(4) - TC(3) = (14(4) + 2000) - (14(3) + 2000) = 14$$

$$MR(3) = TR(4) - TR(3) = (-12(4)^2 + 600(4)) - (-12(3)^2 + 600(3))$$

$$= 2208 - 1692 =$$

$$MR(3) = \underline{516} \text{ dollars/Thing}$$

$$MC(3) = \underline{14} \text{ dollars/Thing}$$

(c) Find the largest interval over which Total Revenue is greater than or equal to \$1200.
(Round answers to the nearest Things)

$$-12x^2 + 600x = 1200$$

$$\Rightarrow 0 = 12x^2 - 600x + 1200$$

$$x = \frac{600 \pm \sqrt{(600)^2 - 4(12)(1200)}}{24}$$

$$x = \frac{600 \pm 549.9090834}{24}$$

$$x = 2.087121525, \text{ and}$$

$$x = 47.91287847$$

$$x = \underline{2} \text{ to } x = \underline{48} \text{ Things}$$

(d) What selling price leads to the largest possible profit?

$$\text{PROFIT} = (-12x^2 + 600x) - (14x + 2000)$$

$$\text{PROFIT} = -12x^2 + 586x - 2000$$

$$x = -\frac{586}{2(-12)} = 24.41\bar{6} \approx 24$$

$$\text{OR } MR(x) = MC(x)$$

$$-24x + 586 = 14$$

$$574 = 24x$$

$$x = 23.91\bar{6}$$

$$\text{price} = -12(\underline{307}) + 600 = -12(\underline{312}) + 600$$

$$-12(\underline{312.99}) + 600$$

$$p = \underline{\$312} \text{ dollars/Thing}$$

4. (10 pts) The average cost of producing x **thousand** items is given by

$$AC(x) = 0.01x^2 - 0.9x + 80 + \frac{20}{x} \quad \text{and} \quad MC(x) = 0.03x^2 - 1.8x + 80,$$

where $AC(x)$ and $MC(x)$ are in dollars/item.

In addition, the selling price per item is a constant $p = 84$ dollars/item.

(a) Give the formulas/values for all the following:

i. Average Variable Cost:

$$AVC(x) = \frac{0.01x^2 - 0.9x + 80}{x} \text{ dollars/item}$$

ii. Total Revenue:

$$TR(x) = 84x \text{ thousand dollars}$$

iii. Marginal Revenue:

$$MR(x) = 84 \text{ dollars/item}$$

(b) Find the minimum value of the marginal cost function.

$$x = -\frac{-1.8}{2(0.03)} = 30$$

$$\begin{aligned} MC(30) &= 0.03(30)^2 - 1.8(30) + 80 \\ &= 53 \end{aligned}$$

$$\text{minimum } MC \text{ value} = 53 \text{ dollars/item}$$

(c) Find the quantity at which profit is maximized.

$$MR(x) \stackrel{?}{=} MC(x)$$

$$84 = 0.03x^2 - 1.8x + 80$$

$$0 = 0.03x^2 - 1.8x - 4$$

$$x = \frac{1.8 \pm \sqrt{(1.8)^2 - 4(0.03)(-4)}}{0.06}$$

$$x = \frac{1.8 \pm 1.928730152}{0.06}$$

$$x = 62.14550254 \text{ or}$$

$$x = -2.14550253$$

$$x = 62.146 \text{ thousand items}$$

5. (10 points) Your company makes two kinds of smoothie mixes: Veri-Tasty and Yum-Drink.

Each pound of Veri-Tasty brings in \$3 dollars in profit and you have enough supplies to make at most 4000 pounds of Veri-Tasty.

Each pound of Yum-Drink brings in \$2.50 dollars in profit and you have enough supplies to make at most 5000 pounds of Yum-Drink.

In total, you can produce and package at most 7000 pounds of mixes.

Let x = the pounds of Veri-Tasty mix and y = pounds of Yum-Drink mix.

(a) Give the constraints, then sketch and shade the feasible region.

You must label all x -intercepts, y -intercepts, and intersection points for full credit.

	x	y	TOTAL
PROFIT	3	2.5	$3x + 2.5y = P(x,y)$

CONSTRAINTS:

$$x \leq 4000$$

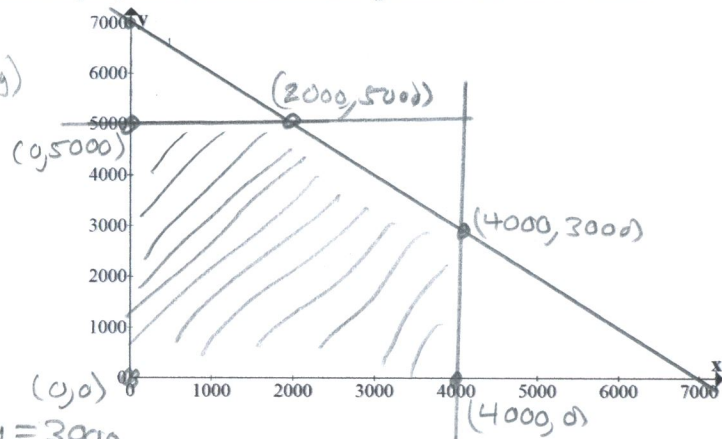
$$y \leq 5000$$

$$x + y \leq 7000$$

INTERSECTIONS:

$$x = 4000 \text{ and } x + y = 7000 \Rightarrow y = 3000$$

$$y = 5000 \text{ and } x + y = 7000 \Rightarrow x = 2000$$



(b) How much of each type of mix should you produce to give maximum profit?

Also give the value of maximum profit? (Show your work)

$$P(x,y) = 3x + 2.5y$$

$$P(0,0) = \$0$$

$$P(0,5000) = \$12,500$$

$$P(4000,0) = \$12,000$$

$$P(4000,3000) = \$19,500$$

$$P(2000,5000) = \$18,500$$

$x =$	<u>4000</u>	pounds of Veri-Tasty
$y =$	<u>3000</u>	pounds of Yum-Drink
Max Profit =	<u>\$19,500</u>	dollars

6. (9 pts) The demand function for a product is given by $\frac{173 - 4p}{q} = 1$, where p is the price per item, in dollars/item, and q in the number of items.

The supply function is **linear**. Suppliers produce 10 items if the price is 25 dollars/item and produce 20 items if the price is 40 dollars/item.

- (a) Find the supply curve. (Write your answer in the form $p = mq + b$).

$$m = \frac{40 - 25}{20 - 10} = \frac{15}{10} = 1.5$$

$$p = 1.5(q - 10) + 25 = 1.5q - 15 + 25$$

$$p = 1.5q + 10$$

- (b) Find the price and quantity that correspond to market equilibrium.

$$\text{COMBINE: } \frac{173 - 4(1.5q + 10)}{q} = 1$$

$$\Rightarrow 173 - 6q - 40 = q$$

$$\Rightarrow 133 = 7q$$

$$\Rightarrow q = 19$$

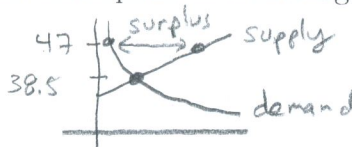
$$p = 1.5(19) + 10 = 38.5$$

$$q = 19 \text{ items}$$

$$p = 38.50 \text{ dollars/item}$$

- (c) Does a market price of \$47 per item correspond to a shortage or surplus?

$$47 > 38.50$$



Circle one: Shortage or Surplus

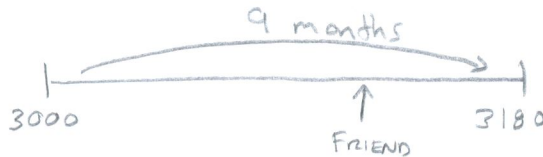
7. (10 pts)

(a) Bill bought a \$3000, 9-month certificate of deposit (CD) that will earn 8% annual simple interest. Three months before the CD was due to mature, Bill needed his CD money, so a friend agreed to lend him money and receive the value of the CD when it matured.

i. Find the value of the CD when it matures.

$$P = 3000, r = 0.08, t = \frac{9 \text{ mo}}{12} = 0.75 \text{ years}$$

$$F = P(1 + rt) = 3000(1 + 0.08(0.75)) = 3180$$



3180 dollars

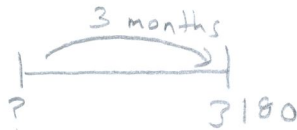
ii. If their agreement allowed the friend to earn a 10% annual simple interest return on his loan to Bill, how much did Bill receive from his friend?

(Round to the nearest cent)

$$F = 3180, r = 0.10, t = \frac{3 \text{ months}}{12} = 0.25 \text{ years}$$

$$3180 = P(1 + 0.10(0.25)) \Rightarrow 3180 = P \cdot 1.025$$

$$P = \frac{3180}{1.025} \approx 3102.439$$



3102.44 dollars

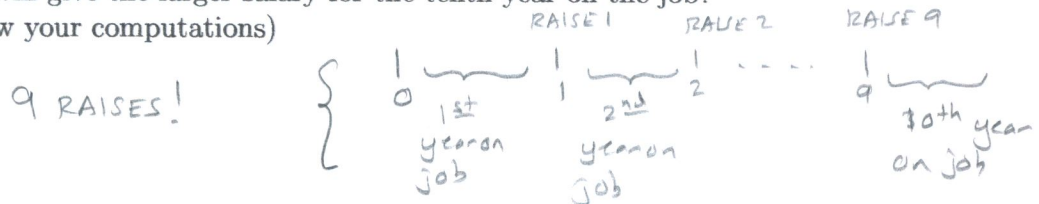
(b) Your boss offers you two salary options.

Option 1: Start with a salary of \$40,000 for the first year on the job and you get raises of \$2,500 at the end of each year.

Option 2: Start with a salary of \$40,000 for the first year on the job and you get 5% raises (compounded) each year.

Which option will give the larger salary for the tenth year on the job?

(You must show your computations)



OPTION 1: $F = 40000 + 2500t = 40000 + 2500(9) = 62,500$

OPTION 2: $F = 40000(1.05)^t = 40000(1.05)^9 = 62,053.13$

Circle one: Option 1 or Option 2

8. (11 points) (Round final answers to two digits after the decimal point).

- (a) You invests \$5000 into an account that has a 6.2% annual rate, compounded continuously. How much total interest do you earn in 4 years?

$$P = 5000, r = 0.062, t = 4$$

$$F = 5000 e^{0.062(4)} \approx \$6407.30$$

$$\text{INTEREST} = 6407.30 - 5000 = 1407.30$$

1407.30 dollars

- (b) You invest \$1000 into an account that pays interest compounded semi-annually. The value in 8 years is \$1650, what is the semi-annual interest rate?

$$P = 1000, r = ?, m = 2, t = 8, F = 1650$$

$$1650 = 1000 \left(1 + \frac{r}{2}\right)^{2(8)}$$

$$1.65 = \left(1 + \frac{r}{2}\right)^{16}$$

$$(1.65)^{1/16} = 1 + \frac{r}{2}$$

$$1.031793402 = 1 + \frac{r}{2}$$

$$0.031793402 = \frac{r}{2}$$

$$0.063586804 = r$$

6.36 %

- (c) You invest \$615 into an account paying 7.3%, compounded monthly. How long does it take to double your investment?

$$P = 615, r = 0.073, m = 12, F = 1230 \quad t = ?$$

$$1230 = 615 \left(1 + \frac{0.073}{12}\right)^{12t}$$

$$2 = (1.00608\bar{3})^{12t}$$

$$\ln(2) = 12t \ln(1.00608\bar{3})$$

$$t = \frac{\ln(2)}{12 \ln(1.00608\bar{3})} \approx 9.524019316 \text{ year}$$

9.52 years

9. (12 points) (Round final answers to two digits after the decimal point).

- (a) Fred wants to make regular payments to save up \$150,000 by the time his daughter, Pebbles, turns 18 years old. His account earns 5% interest, compounded quarterly. How much must he deposit into the account at the end of each quarter after Pebbles is born to reach his goal?

$$R = ? , F = 150000 , t = 18 , r = 0.05 , m = 4$$

$$i = \frac{0.05}{4} = 0.0125 , n = 4(18) = 72$$

$$150000 = R \frac{(1.0125)^{72} - 1}{0.0125}$$

$$150000 = R \cdot 115.6736215$$

$$R \approx 1296.75$$

1296.75 dollars

- (b) What amount must be set aside now to generate payments of \$40,000 at the beginning of each year for the next 14 years if the account gets 5.5%, compounded annually?

$$P = ? , R = 40000 , r = 0.055 , m = 1 , t = 14$$

$$i = \frac{r}{m} = 0.055$$

$$n = mt = 14$$

$$P = 40000 \frac{1 - (1.055)^{-14}}{0.055} (1.055)$$

$$P \approx 404683.1412$$

404683.14 dollars

- (c) When Pebbles graduates from college, her student loans total \$24,000.

These loans are at 3%, compounded monthly and are to be paid off with payments at the end of each month for the next 10 years. How much are the monthly payments?

$$P = 24000 , R = ? , r = 0.03 , m = 12 , t = 10$$

$$i = \frac{0.03}{12} = 0.0025 , n = 12 \cdot 10 = 120$$

$$24000 = R \frac{1 - (1.0025)^{-120}}{0.0025}$$

$$24000 = R \cdot 103.5617531$$

$$R \approx 231.7457672$$

231.75 dollars