1. (12 points)

(a) Suppliers are willing to produce 56 items if the price is \$440/item and 136 items if the price is \$530/item. The supply curve is **linear**.

i. Give the equation for the supply curve. (Use p for price and q for quantity).

$$m = \frac{530-440}{136-56} = \frac{90}{80} = 1.125$$

$$p = 1.125(q-56)+440$$

$$p = 1.125q + 377$$

$\boxed{p = 1.125q + 377}$

ii. You are also told that the demand curve is $2p + 6q = 1447$.
Find the quantity and price that corresponds to market equilibrium.

INTERSECT!

$$2(1.125q+377)+6q = 1447$$

$$2.25q + 754 + 6q = 1447$$

$$8.25q = 693$$

$$q = 84$$

$$p = 1.125(84)+377 = 471.50$$

$q =$ 84 ~~toys~~ item

$p =$ 471.50 dollars/~~toy~~ item

(b) Solve $3(1+4e^{0.1x}) = 27$ for x (give your final answer accurate to 3 digits after the decimal).

$$1+4e^{0.1x} = 9$$

$$4e^{0.1x} = 8$$

$$e^{0.1x} = 2$$

$$0.1x = \ln(2)$$

$$x = \frac{\ln(2)}{0.1} \approx 6.931471806$$

$\boxed{x = 6.931}$

2. (12 points) You are given average variable cost and marginal cost for a product:

$$AVC(x) = x^2 - 3.4x + 7 \text{ dollars/item} \quad \text{and} \quad MC(x) = 3x^2 - 6.8x + 7 \text{ dollars/item},$$

where x is in thousands of items. You also know that fixed cost is $FC = 2$ thousand dollars. Round your final answers to the nearest item or nearest cent.

(a) Find and simplify the formulas for total cost and average cost.

$VC(x) = x\,AVC(x) = x(x^2 - 3.4x + 7) = x^3 - 3.4x + 7x$

$TC(x) =$ $x^3 - 3.4x^2 + 7x + 2$ thousand dollars

$AC(x) =$ $x^2 - 3.4x + 7 + \frac{2}{x}$ dollars/item

(b) Find the shutdown price.

LOWEST y-VALUE OF AVC(x)

OR solve $AVC(x) = MC(x)$ to get this

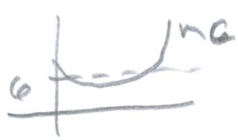
$x = -\frac{-3.4}{2(1)} = 1.7$

$AVC(1.7) = (1.7)^2 - 3.4(1.7) + 7 = 4.11$

$SDP =$ $4.\underline{11}$ dollars/item

(c) Find the range of quantities over which $MC(x)$ is less than or equal to \$6 per item.

$MC(x) \stackrel{?}{=} 6 \Rightarrow 3x^2 - 6.8x + 7 = 6$



$3x^2 - 6.8x + 1 = 0$

$x = \frac{6.8 \pm \sqrt{6.8^2 - 4(3)(1)}}{2(3)}$

$x = \frac{6.8 \pm \sqrt{34.24}}{6} \approx \frac{6.8 \pm 5.851495535}{6}$

$x \approx 0.158084078$ to $x \approx 2.108582589$

$x =$ 0.158 to $x =$ 2.109 thousand items

3. (14 points) The total cost to produce x tennis balls is given by: $TC(x) = 0.5x + 42$ dollars.
The price per ball for an order of x tennis balls is given by: $p = 4 - 0.05x$ dollars/ball.

(a) Find the quantity at which Average Cost is equal to \$7.50 per ball.

$$AC(x) = \frac{0.5x+42}{x} = 0.5 + \frac{42}{x} \overset{?}{=} 7.50$$

$$\Rightarrow \quad 0.5x + 42 = 7.5x$$

$$42 = 7x$$

$$x = 6$$

$\boxed{x = \underline{\quad 6 \quad} \text{ balls}}$

(b) Find and simplify the formulas for Total Revenue and Marginal Revenue.
(Recall: $MR(x) = TR(x+1) - TR(x)$).

$$TR(x) = (\text{price})\,x = (4 - 0.05x)x = 4x - 0.05x^2$$

$$MR(x) = \left[4(x+1) - 0.05(x+1)^2\right] - \left[4x - 0.05x^2\right]$$

$$= \left[4x + 4 - 0.05(x^2 + 2x + 1)\right] - \left[4x - 0.05x^2\right]$$

$$= \cancel{4x} + 4 \cancel{- 0.05x^2} - 0.1x - 0.05 \cancel{- 4x} \cancel{+ 0.05x^2}$$

$$= -0.1x + 3.95$$

$TR(x) = \underline{\quad 4x - 0.05x^2 \quad}$ dollars

$MR(x) = \underline{\quad -0.1x + 3.95 \quad}$ dollars/ball.

(c) Find the price that corresponds to maximum profit.

$$\text{PROFIT} = TR(x) - TC(x) = \left[4x - 0.05x^2\right] - \left[0.5x + 42\right]$$

$$P(x) = -0.05x^2 + 3.5x - 42$$

$$x = -\frac{3.5}{2(-0.05)} = 35$$

$$p = 4 - 0.05(35) = 2.25$$

$\boxed{p = \underline{\quad 2.25 \quad} \text{ dollars/ball.}}$

4. (12 points) Your company makes two kinds of soda: Regular and Diet.

Your total daily production of soda is limited to 1000 gallons.

Production requires 2 cup of sugar per gallon of Regular and $\frac{1}{2}$ cup of sugar per gallon of Diet. Today, you are limited to 626 cups of sugar.

The profit is \$1 per gallon of Regular soda and \$1.20 per gallon of Diet soda.

Let x = the gallons of Regular soda and y = the gallons of Diet soda that you produce and sell.

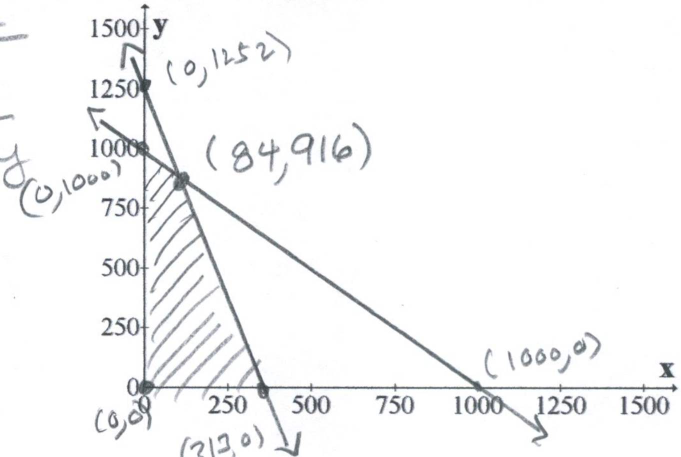
(a) Give the constraints, then sketch and shade the feasible region.
You **must** label ALL x-intercepts, y-intercepts, and **intersection points** for full credit.

	x	y	TOTAL
SUGAR	2	$\frac{1}{2}$	$2x+\frac{1}{2}y$
PROFIT	1	1.20	$x+1.20y$

CONSTRAINTS:

$x+y \le 1000$ (0,1000) (1000,0)

$2x+\frac{1}{2}y \le 626$ (0,1252) (313,0)



INTERSECT:

$x+y=1000 \Rightarrow y=1000-x$

$2x+\frac{1}{2}(1000-x)=626$

$2x+500-\frac{1}{2}x=626$

$1.5x=126 \quad x=84 \Rightarrow y=1000-84=916$

(b) How much of each type of soda should you produce to give maximum profit?
Also give the value of maximum profit? (Show your work)

$P(0,0)=0$

$P(0,1000)=(0)+1.20(1000)=1200$

$P(313,0)=(313)+1.20(0)=313$

$P(84,916)=(84)+1.20(916)=1183.20$

x = 0 gallons of Regular soda

y = 1000 gallons of Diet soda

Max Profit = 1200 dollars