MATH 111
Final Exam
Autumn 2021

Name _____

Student ID #_____ Section _____

HONOR STATEMENT

"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

SIGNATURE:_____

- Check that your exam contains **6** problems.
- You are allowed to use a non-graphing scientific calculator, a ruler, and one 100-page notebook containing hand-written notes. All other sources are forbidden.
- Turn your cell phone OFF and put it away for the duration of the exam.
- You may not listen to headphones or earbuds during the exam.
- **You must show your work.** Clearly label lines and points that you are using and show all calculations. The correct answer with no supporting work may result in no credit.
- If you use a guess-and-check method when an algebraic method is available, you may not receive full credit.
- When rounding is necessary, you may round your final answer to two digits after the decimal.
- **Do not write within 1 centimeter of the edge!** Your exam will be scanned for grading.
- If you run out of room, write on the last page and **indicate that you have done so**.
- There are multiple versions of the exam, you have signed an honor statement, and cheating is a hassle for everyone involved. DO NOT CHEAT.

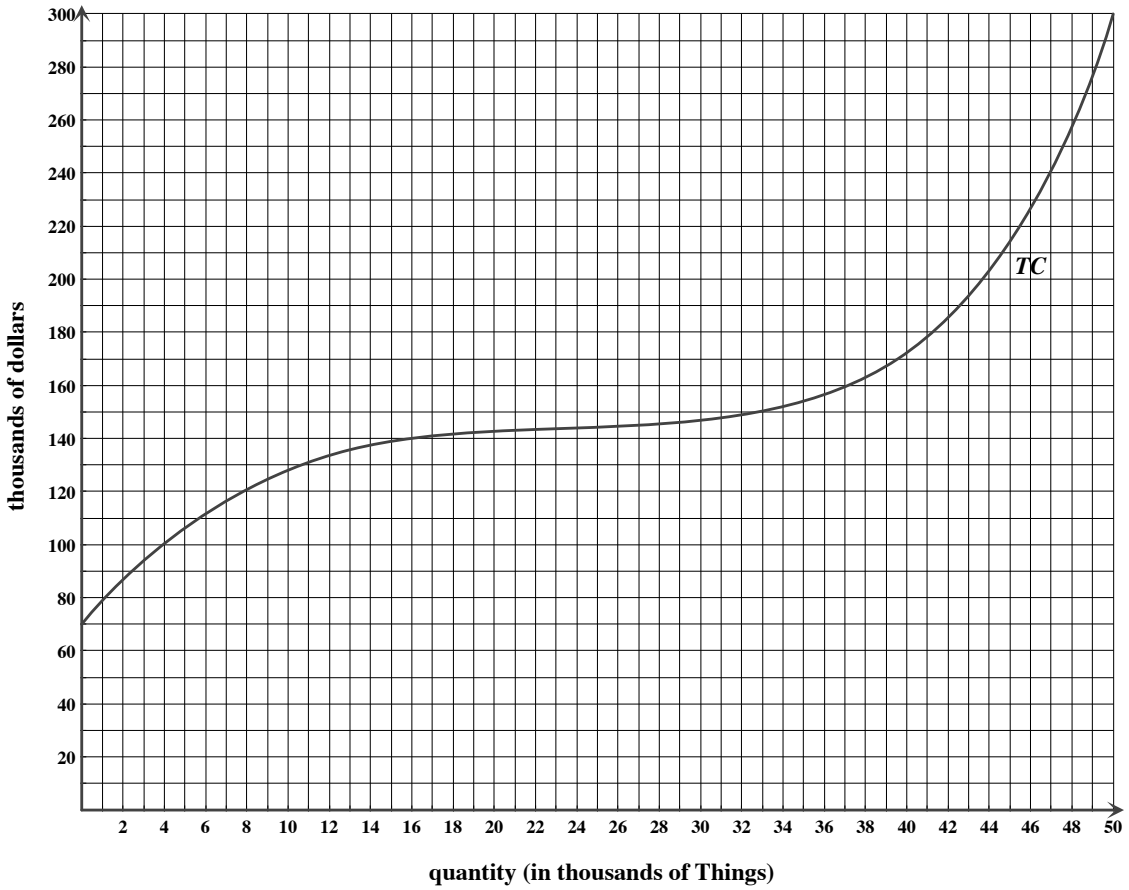
GOOD LUCK!

Suppose you produce and sell Things. The following table summarizes the terms we've learned so far relating to revenue and cost. Assume you are given a graph of total cost $TC(q)$ and total revenue $TR(q)$ for producing and selling q Things.

Term	**Definition**	**Related equations and formulas**	**Graphical Interpretation**
total cost $TC(q)$	the total amount you spend to produce q Things	$TC(q) = VC(q) + FC$	—
variable cost $VC(q)$	the money you spend to produce q Things without including fixed costs	$VC(q) = TC(q) - FC$	the graph of VC has the same shape as TC and goes through the origin
fixed cost FC	the money you must spend even if you produce 0 Things; also known as *overhead*	$FC = TC(q) - VC(q)$ $FC = TC(0)$	the vertical distance between the TC and VC graphs OR the "y"-intercept of the TC graph
average cost $AC(q)$	total cost averaged over the number of Things produced	$AC(q) = \frac{TC(q)}{q}$	the slope of the diagonal line through the TC graph at q
average variable cost $AVC(q)$	variable cost averaged over the number of Things produced	$AVC(q) = \frac{VC(q)}{q}$	the slope of the diagonal line through the VC graph at q
breakeven price BEP	the smallest value of average cost	—	the slope of the least steep diagonal line that intersects the TC graph
shutdown price SDP	the smallest value of average variable cost	—	the slope of the least steep diagonal line that intersects the VC graph
marginal cost $MC(q)$ (see footnote)	the incremental rate of change in TC from q to $q+1$ Things	$MC(q) = \frac{TC(q+1)-TC(q)}{1}$	the slope of the secant line through TC (or VC) at q and $q+1$
total revenue $TR(q)$	the total amount you receive when you sell q Things	—	—
average revenue $AR(q)$	total revenue averaged over the number of Things sold; also known as *price per Thing*	$AR(q) = \frac{TR(q)}{q}$	the slope of the diagonal line through the TR graph at q
marginal revenue $MR(q)$ (see footnote)	the incremental rate of change in TR from q to $q+1$ Things	$MR(q) = \frac{TR(q+1)-TR(q)}{1}$	the slope of the secant line through the TR graph at q and $q+1$
profit $P(q)$	the money you are left with after subtracting total cost from total revenue	$P(q) = TR(q) - TC(q)$	the vertical distance between TR and TC (when $TR > TC$)

NOTE: If q is measured in hundreds or thousands of Things, the definitions, formulas, and graphical interpretations of marginal revenue and marginal cost must be adjusted appropriately.

1. (15 points) The following is the graph of total cost (in **thousands of dollars**) for selling q **thousand things**.



(a) What is the value of fixed cost?

ANSWER: $FC =$_____thousand dollars

(b) What is the value of variable cost at $q = 47$ thousand things?

ANSWER: $VC(47) =$_____thousand dollars

(c) At what quantity is average cost (AC) 13 dollars per Thing?

ANSWER: $q =$_____thousand Things

(d) Things sell for \$9 each. Sketch the graph of TR on the axes above and answer the following.

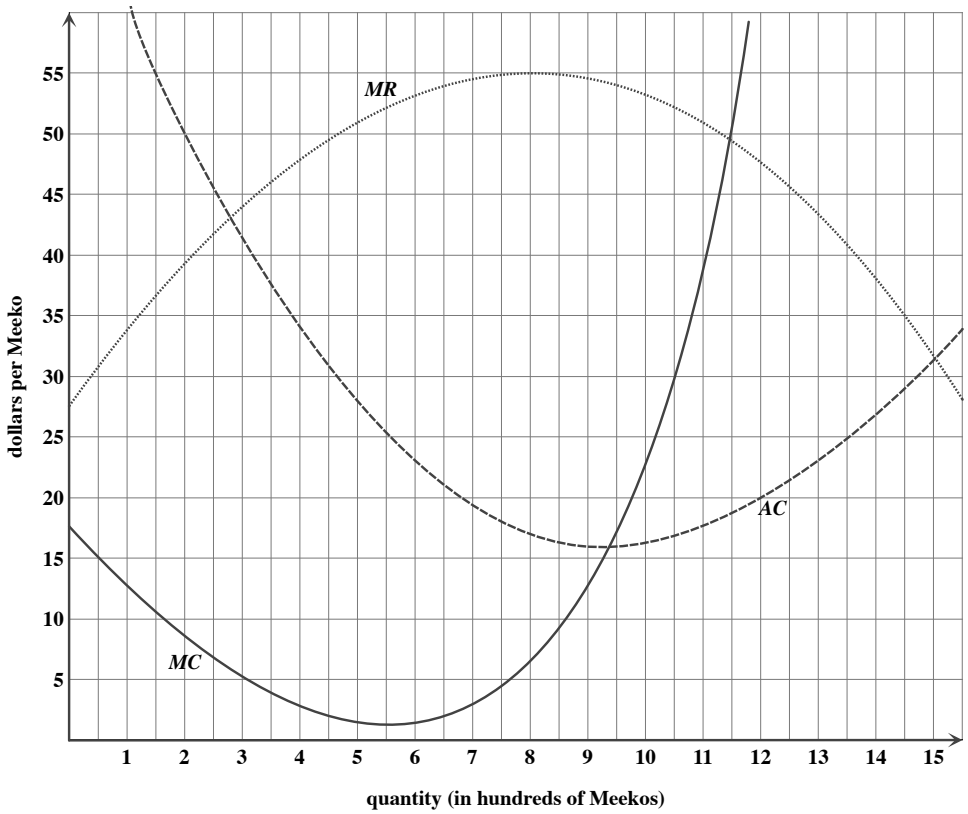
i. What is the smallest quantity at which we're not forced to take a loss?

ANSWER: $q =$_____thousand Things

ii. What quantity maximizes profit?

ANSWER: $q =$_____thousand Things

2. (15 points) The graphs below are marginal cost (MC), average cost (AC), and marginal revenue (MR) for producing and selling Meekos.



(a) What is the largest value of **marginal revenue**?

ANSWER: _____dollars per Meeko

(b) What is the change in **total cost** if quantity increases from 400 to 401 Meekos?

ANSWER: _____dollars

(c) What is the **total cost** to produce 6 hundred Meekos?

ANSWER: _____hundred dollars

(d) Which can you read from this graph: the breakeven price or the shutdown price? What is its value?

ANSWER: (circle one) breakeven price shutdown price

_____dollars per Meeko

(e) What quantity maximizes profit?

ANSWER: q =_____hundred Meekos

3. (20 points) You sell Things. The formula for total cost is

$$TC(q) = 0.1q^3 - 5q^2 + 90q + 24,$$

where q is in **hundreds of Things** and TC is in **hundreds of dollars.**

(a) Compute the **average cost** to produce 2 hundred Things. Include units with your answer.

ANSWER: _____UNITS: _____

(b) Give formulas for **variable cost** and **average variable cost** for selling q hundred Things.

ANSWER: $VC(q) =$_____

$AVC(q) =$_____

(c) Find all values of q at which **average variable cost** is 35 dollars per Thing. (Round your final answers to two digits after the decimal.)

ANSWER: (list all) $q =$_____hundred Things

(d) Compute the shutdown price. (Round to the nearest cent.)

ANSWER: _____dollars per Thing

(e) The graph of total revenue is a straight line and **profit** is 0 when $q = 20$ hundred Things. Find the formula for $TR(q)$.

ANSWER: $TR(q) =$_____

4. (15 points) Consider the two functions,

$$f(x) = 5x - x^2 \quad \text{and} \quad g(x) = 3x^2 - 4x + 5.$$

Round all your final answers to two digits after the decimal.

(a) Find the slope of the diagonal line to $g(x)$ at $x = 2$.

ANSWER: _____

(b) Find and *completely simplify* $\dfrac{f(1+h) - f(1)}{h}$.

ANSWER: $\dfrac{f(1+h) - f(1)}{h}$ = _____

(c) Find largest value of x at which the graphs of $f(x)$ and $g(x)$ intersect.

ANSWER: x = _____

(d) Find the longest interval of values of x over which $f(x)$ and $g(x)$ are both increasing.

ANSWER: from x = _____ to x = _____

5. (20 points) *For all your work below, round your **final answer** to two digits after the decimal*

(a) Grover invests \$3,000 in a bank account that pays simple interest. After 5 years, the account has earned \$1,215 in total interest. What is the annual interest rate on the account?

ANSWER: _____%

(b) Abby found an investment that will pay her 5% annual interest, compounded quarterly. How much must Abby invest in the account now so that she will have \$10,000 in five years?

ANSWER: _____dollars

(c) Elmo deposits \$600 into an account that pays 4% annually, compounded continuously. How long will it take for the account balance to triple?

ANSWER: _____years

(d) Oscar buys a home for \$320,000. Six years later, he sells the home for \$400,000. What interest rate, compounded annually, did this investment represent for Oscar?

ANSWER: _____%

6. (15 points)

(a) Ernie makes regular payments of \$500 at the beginning of every six-month period into an account that earns 4% annually, compounded semi-annually. After how many semi-annual payments will the balance in the account first exceed \$6,000? (Round your final answer UP to the nearest whole number of payments).

ANSWER: _____payments

(b) Samantha buys a car with the help of a loan. The car costs \$35,000 and she makes a down payment of \$5,000. Her loan earns a 9% interest rate, compounded monthly. She will make her first payment at the end of this month and each month afterward for the next 10 years to pay off the entire loan. How big is each payment? (Round your final answer to the nearest cent.)

ANSWER: _____dollars

(c) Bert has \$20,000 saved in an account that earns 6% annually, compounded quarterly. He starts making payments of \$1000 at the end of each quarter into the same type of account. How much money will he have saved up in total in both accounts after 5 years? AND how much total interest did Bert earn in both accounts? (Round your final answers to the nearest cent.)

ANSWERS: Total money in both accounts in 5 years = _____ dollars

Total Interest Earned = _____ dollars

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