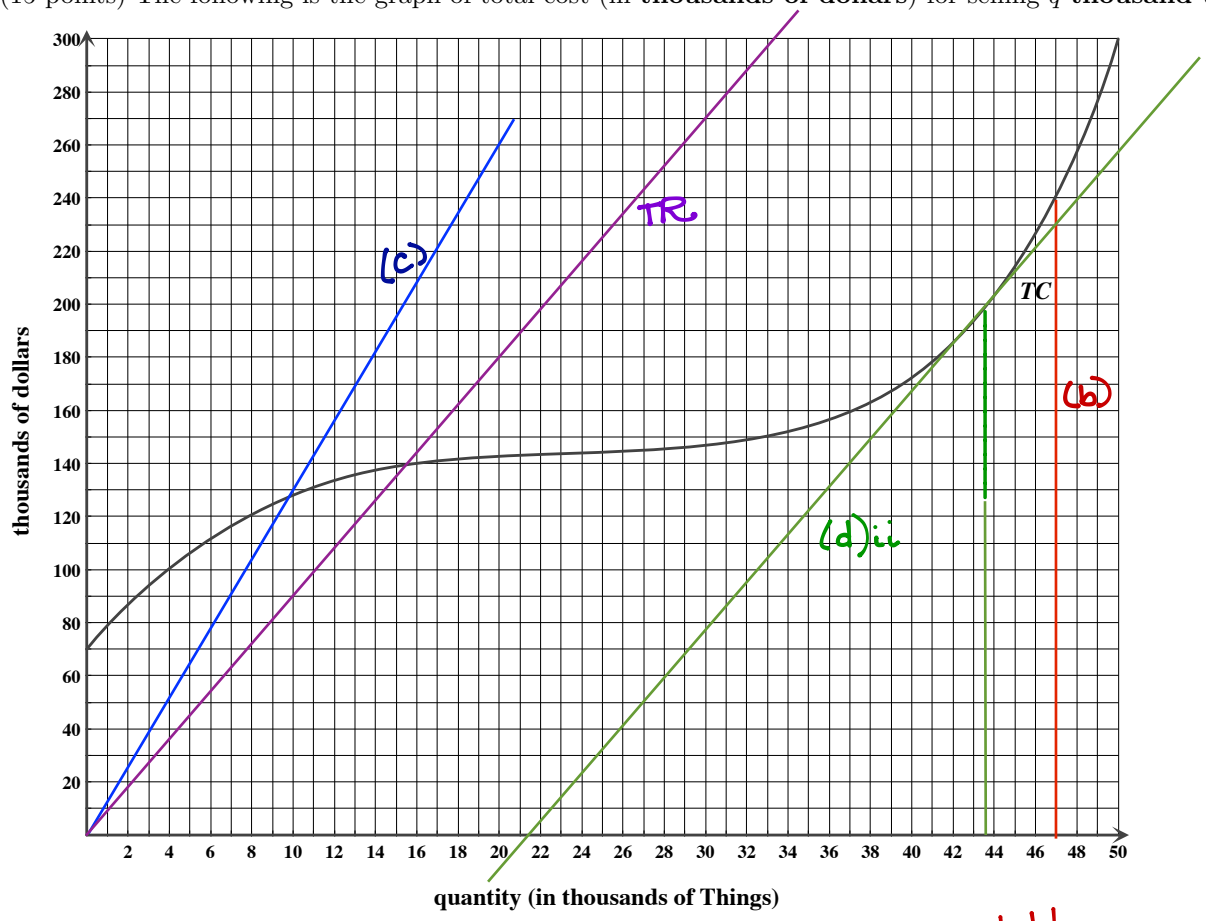
1. (15 points) The following is the graph of total cost (in **thousands of dollars**) for selling q **thousand things**.



(a) What is the value of fixed cost?

acceptable range [70, 75)

ANSWER: $FC =$ 70 thousand dollars

(b) What is the value of variable cost at $q = 47$ thousand things?

$TC(47) \approx 240$

$VC(47) \approx 240 - 70$

(165, 175)

ANSWER: $VC(47) =$ ~170 thousand dollars

(c) At what quantity is average cost (AC) 13 dollars per Thing?

[8.5, 10]

ANSWER: $q =$ ~9.8 thousand Things

(d) Things sell for \$9 each. Sketch the graph of TR on the axes above and answer the following.

i. What is the smallest quantity at which we're not forced to take a loss?

(14, 16)

ANSWER: $q =$ ~15.5 thousand Things

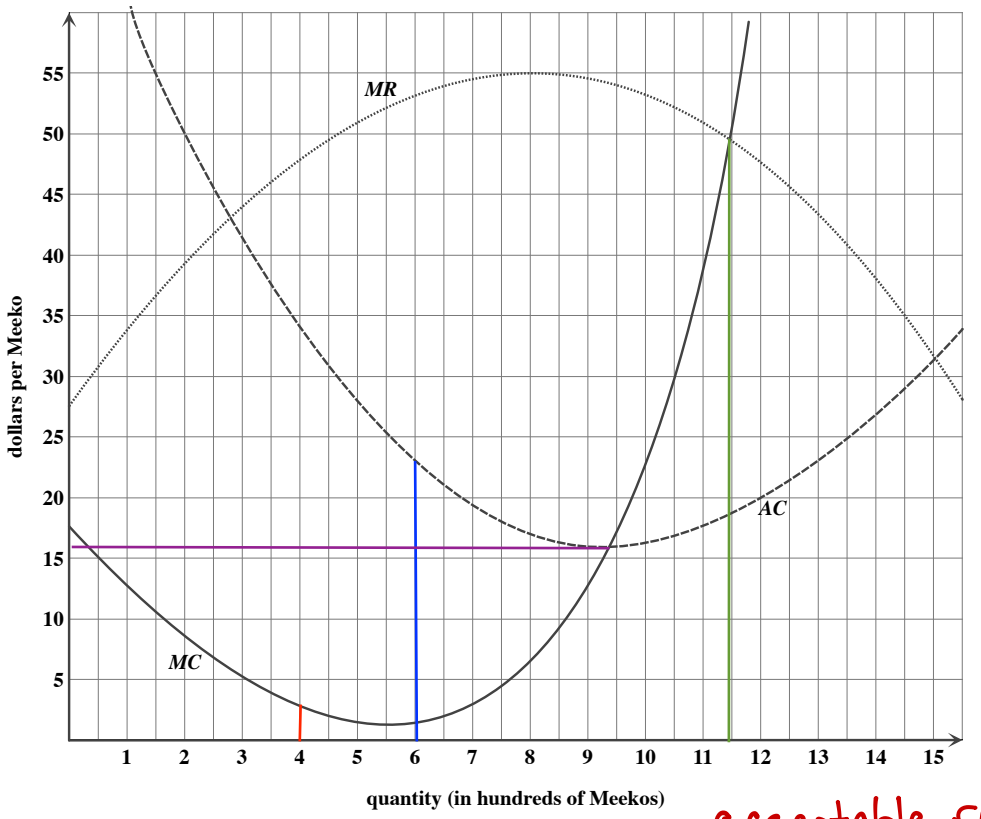
ii. What quantity maximizes profit?

(42, 45)

maybe [ ]?

ANSWER: $q =$ ~43.5 thousand Things

2. (15 points) The graphs below are marginal cost (MC), average cost (AC), and marginal revenue (MR) for producing and selling Meekos.



(a) What is the largest value of **marginal revenue**?

acceptable range
[54, 56)

ANSWER: 55 dollars per Meeko

(b) What is the change in **total cost** if quantity increases from 400 to 401 Meekos?

$MC(4) \approx 2.5$ \$/Meeko

[2, 4]

ANSWER: ~2.5 dollars

(c) What is the **total cost** to produce 6 hundred Meekos?

$AC(6) \approx 23$

$TC(6) \approx 23 \times 6$

[135, 150)

ANSWER: 138 hundred dollars

(d) Which can you read from this graph: the breakeven price or the shutdown price? What is its value?

ANSWER: (circle one) (breakeven price) shutdown price

(15, 17.5)

~16 dollars per Meeko

(e) What quantity maximizes profit?

[11.4, 11.5]

ANSWER: $q =$ ~11.49 hundred Meekos

~4 each

3. (20 points) You sell Things. The formula for total cost is

$$TC(q) = 0.1q^3 - 5q^2 + 90q + 24,$$

where q is in **hundreds of Things** and TC is in **hundreds of dollars.**

(a) Compute the **average cost** to produce 2 hundred Things. Include units with your answer.

$AC(2) = \frac{TC(2)}{2} = \frac{184.8 \text{ hundred \$}}{2 \text{ hundred Things}} = 92.4 \text{ \$/Thing}$

ANSWER: 92.4 UNITS: \$/Thing

(b) Give formulas for **variable cost** and **average variable cost** for selling q hundred Things.

ANSWER: $VC(q) = 0.1q^3 - 5q^2 + 90q$

$AVC(q) = 0.1q^2 - 5q + 90$

(c) Find all values of q at which **average variable cost** is 35 dollars per Thing. (Round your final answers to two digits after the decimal.)

$0.1q^2 - 5q + 90 = 35$

$0.1q^2 - 5q + 55 = 0$

$q = \frac{5 \pm \sqrt{25 - 4(0.1)(55)}}{0.2}$

$= \frac{5 \pm \sqrt{3}}{0.2} = 16.34, 33.66$

ANSWER: (list all) $q =$ 16.34, 33.66 hundred Things

(d) Compute the shutdown price. (Round to the nearest cent.)

$q = \frac{5}{0.2} = 25$

$AVC(25) = 27.5$

ANSWER: 27.50 dollars per Thing

(e) The graph of total revenue is a straight line and **profit** is 0 when $q = 20$ hundred Things. Find the formula for $TR(q)$.

$TR(20) = TC(20) = 624$

$(0,0)(20,624)$

$m = \frac{624 - 0}{20 - 0} = 31.2$

ANSWER: $TR(q) = 31.2q$

Problem 4 Version 1

4. (15 points) Consider the two functions,

$$f(x) = 5x - x^2 \quad \text{and} \quad g(x) = 3x^2 - 4x + 5.$$

Round all your final answers to two digits after the decimal.

3 pts

(a) Find the slope of the diagonal line to $g(x)$ at $x = 2$.

$\frac{g(2)}{2} = \frac{3(2)^2 - 4(2) + 5}{2} = \frac{12 - 8 + 5}{2} = \frac{9}{2}$

+1 +1 +1

ANSWER: 4.5

4 pts

(b) Find and *completely simplify* $\frac{f(1+h) - f(1)}{h}$.

$\frac{[5(1+h) - (1+h)^2] - [5(1) - (1)^2]}{h}$] SIGNIFICANT UNDERSTANDING ERRORS HERE ⇒ −4

$= \frac{5 + 5h - 1 - 2h - h^2 - 4}{h} = \frac{3h - h^2}{h}$ SMALL MISCOPY ERROR −1

MAJOR EXPANSION OR SIMPLIFYING ERROR −2

ANSWER: $\frac{f(1+h) - f(1)}{h} =$ 3 − h

4 pts

(c) Find largest value of x at which the graphs of $f(x)$ and $g(x)$ intersect.

$5x - x^2 = 3x^2 - 4x + 5$ SET UP −2 ERROR

$0 = 4x^2 - 9x + 5$

USING VERTEX −3 →

$x = \frac{9 \pm \sqrt{81 - 4(4)(5)}}{2(4)} = \frac{9 \pm \sqrt{1}}{8}$

$\frac{8}{8} = 1$

$\frac{10}{8} = 1.25$ ← LARGER

SMALL CALCULATION ERROR −1

ANSWER: $x =$ 1.25

4 pts

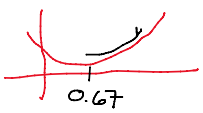
(d) Find the longest interval of values of x over which $f(x)$ and $g(x)$ are both increasing.

$f(x) \Rightarrow \quad x = -\frac{5}{2(-1)} = 2.5$] +1

increasing before 2.5

$g(x) \Rightarrow \quad x = -\frac{(-4)}{2(3)} = \frac{4}{6} = 0.\overline{6}$] +1

increasing after 0.67



CONCLUSION +2

[ANSWER: from $x =$ 0.67 to $x =$ 2.5

Problem 5 Version 1

5. (20 points) *For all your work below, round your* ***final answer*** *to two digits after the decimal*

5 pts

(a) Grover invests $3,000 in a bank account that pays simple interest. After 5 years, the account has earned $1,215 in total interest. What is the annual interest rate on the account?

$B = P(1+rt)$ ← +1

$4215 = 3000(1+r\cdot 5)$] +2

$1.405 = 1+5r$

$0.405 = 5r$

$r = \frac{0.405}{5} = 0.081$] +2

ANSWER: 8.1 %

5 pts

(b) Abby found an investment that will pay her 5% annual interest, compounded quarterly. How much must Abby invest in the account now so that she will have $10,000 in five years?

+1 [$B = P(1.0125)^{4t}$ $\quad \frac{r}{m} = \frac{0.05}{4} = 0.0125$

+2 [$10000 = P(1.0125)^{20}$

+2 [$P = \frac{10000}{(1.0125)^{20}} \approx 7800.085$

ANSWER: 7800.09 dollars

5 pts

(c) Elmo deposits $600 into an account that pays 4% annually, compounded continuously. How long will it take for the account balance to triple?

+1 [$B = 600e^{0.04t}$

+2 [$1800 = 600e^{0.04t}$

$3 = e^{0.04t}$

$\ln(3) = 0.04t$

$t = \frac{\ln(3)}{0.04} \approx 27.465307$] +2

ANSWER: 27.47 years

5 pts

(d) Oscar buys a home for $320,000. Six years later, he sells the home for $400,000. What interest rate, compounded annually, did this investment represent for Oscar?

+1 [$B = P(1+r)^t$

+2 [$400000 = 320000(1+r)^6$

$1.25 = (1+r)^6$

+2 [$(1.25)^{1/6} = 1+r$

$1.03789082 = 1+r$

$r = 0.03789$

ANSWER: 3.79 %

Problem 6 Version 1

6. (15 points)

5 pts

(a) Ernie makes regular payments of \$500 at the beginning of every six-month period into an account that earns 4% annually, compounded semi-annually. After how many semi-annual payments will the balance in the account first exceed \$6,000? (Round your final answer UP to the nearest whole number of payments).

+1 $F = R\,\frac{(1+i)^n - 1}{i}\,(1+i)$

+1 $6000 = 500\,\frac{(1.02)^n - 1}{0.02}\,(1.02)$

$\Rightarrow 0.23529412 = (1.02)^n - 1$

+1 $1.23529412 = (1.02)^n$

$i = \frac{0.04}{2} = 0.02$

$\ln(1.23529412) = n \ln(1.04)$ +1

$\Rightarrow n = \frac{\ln(1.23529412)}{\ln(1.04)}$

≈ 10.6708 PAYMENTS +1

ANSWER: 11 payments

5 pts

(b) Samantha buys a car with the help of a loan. The car costs \$35,000 and she makes a down payment of \$5,000. Her loan earns a 9% interest rate, compounded monthly. She will make her first payment at the end of this month and each month afterward for the next 10 years to pay off the entire loan. How big is each payment? (Round your final answer to the nearest cent.)

+1 $P = R\,\frac{1 - (1.0075)^{-n}}{0.0075}$ $\quad i = \frac{0.09}{12} = 0.0075$

$n = 12 \cdot 10 = 120$

+2 $30{,}000 = R \cdot 78.941692669$

+2 $R = 380.02732$

ANSWER: 380.03 dollars

5 pts

(c) Bert has \$20,000 saved in an account that earns 6% annually, compounded quarterly. He starts making payments of \$1000 at the end of each quarter into the same type of account. How much money will he have saved up in total in both accounts after 5 years? AND how much total interest did Bert earn in both accounts? (Round your final answers to the nearest cent.)

+1 $B = 20000\,(1.015)^{20} = 26937.10$

+1 $F = 1000\,\frac{(1.015)^{20} - 1}{0.015} = 23{,}123.67$

+1 ADD TOGETHER

CONTRIBUTIONS $= 20{,}000 + 1000 \cdot 20 = 40{,}000$

ANSWERS: Total money in both accounts in 5 years = 50,060.77 dollars

Total Interest Earned = 10,060.77 dollars

+2