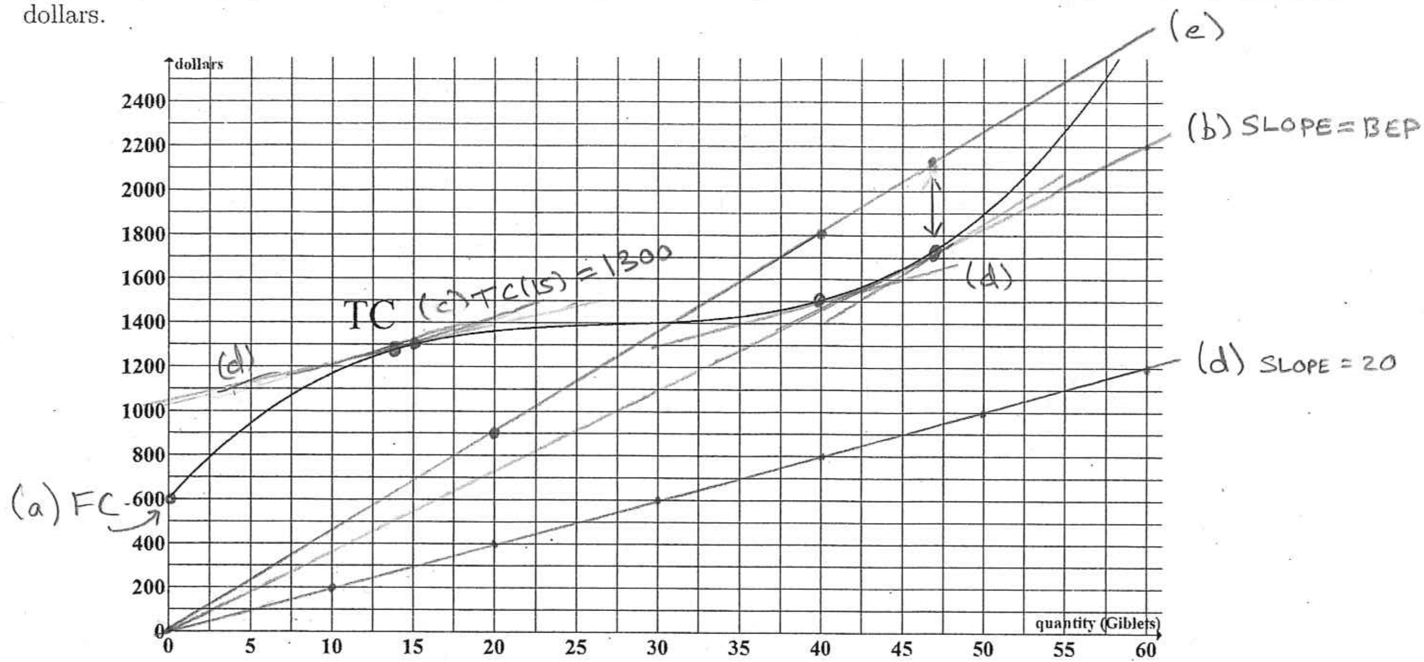
V1

1. (20 points) The graph of **total cost** for producing Giblets are given. The x-axis is in Giblets and the y-axis in in dollars.



Make sure to **read the description above the graph** before you do the problems! Show and label your work in the graph. Round your estimates to the nearest cent or nearest Giblet.

4 pts (a) What is the value of fixed costs?

$TC(0) = FC = 600$

OKAY IF THEY SAY A VALUE CLOSE (RANGE: 600 TO 610)

$FC =$ ~600 dollars

4 pts (b) Find the **Break Even Price** (BEP).

TWO POINTS: (0,0) (60, 2200) $\Rightarrow$ SLOPE $= \frac{2200-0}{60-0} = 36.\overline{6}$

RANGE: $\frac{2150}{60} = 35.83$ TO $\frac{2250}{60} = 37.50$

NOTE: THE LINE HITS THE GRAPH AROUND $x = 44$ −3 FOR THIS ANSWER

$BEP =$ ~36.67 dollars per Giblet

4 pts (c) Find the **average variable cost** at $q = 15$ Giblets.

$\underbrace{AVC(15) = \frac{VC(15)}{15} = \frac{TC(15) - FC}{15}}_{+1 \text{ FOR SOME KNOWLEDGE OF FORMULAS}} = \frac{1300-600}{15} = \frac{700}{15} = 46.\overline{6}$

RANGE: $\frac{675}{15} = 45$ TO $\frac{725}{15} = 48.\overline{3}$

$AVC(15) =$ ~46.67 dollars per Giblet

4 pts (d) Give the longest interval of quantities over which marginal cost is **at most** 20 dollars per tablet.

SLOPE = 20 $\Rightarrow$ (0,0) (10, 200) (20, 400), (30, 600), ..., (60, 1200)

FIND WHEN TANGENT SLOPE = 20

COMMON ERROR: EXTENDING A TANGENT & USING OTHER INTERSECTION −2 FOR THIS, LIKE 14 TO 55 OR 2 TO 40

RANGE: 13 TO 17 RANGE: 38 TO 42

from $q =$ ~14 to $q =$ ~40 Giblets

4 pts (e) Suppose the market price is \$45.00 per Giblet. Find the quantity that maximizes profit and give the value of maximum profit.

TR line (0,0), (20, 900), (40, 1800)

RANGE 45.5 TO 48

TR(47) − TC(47) = 2125 − 1725 = 400

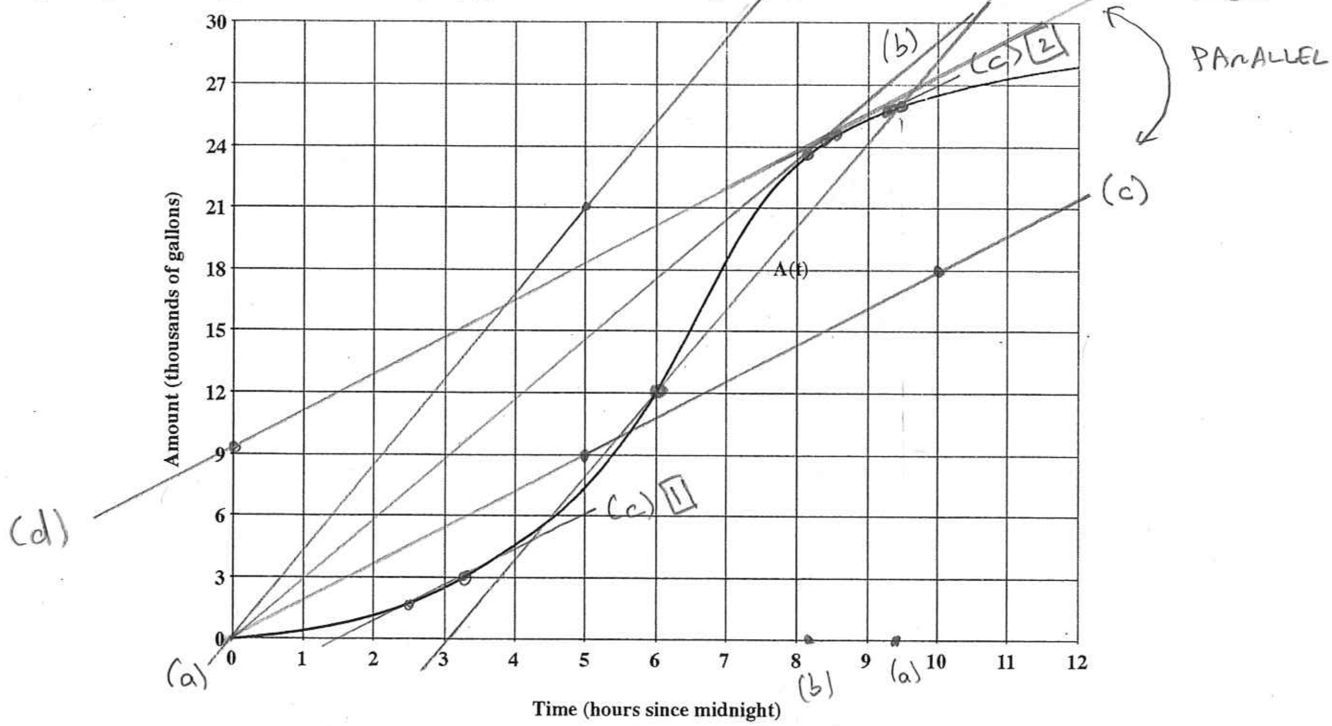
RANGE 350 TO 450

$q =$ ~47 Giblets and Profit = ~400 dollars

VI

2. (16 points) The graph below gives the amount of water, $A(t)$, that flows out of a reservoir over a 12-hour period beginning at midnight. The amount, $A(t)$, is in thousands of gallons and the time t is in hours after midnight.

PARALLEL



Show and label your work in the graph.

4pts (a) Find a value of t, larger than 6, such that $\dfrac{A(t)-A(6)}{t-6} = 4.2$.

SLOPE = 4.2 (0,0) (5,21)

+1 FOR REFERENCE LINE

$t-6$ START = 6 END = t

RANGE 9.1 TO 9.7

$t =$ ~9.4 hours

4pts (b) Find the time when the overall rate of flow out of the reservoir is largest.

LARGEST DIAGONAL SLOPE

−2 IF THEY GIVE THE SLOPE OF THIS LINE

RANGE 7.9 TO 8.3

$t =$ ~8.1 hours

4pts (c) During how many one-hour intervals is water flowing out at an average rate of 1.8 thousand gallons per hour?

+1. FOR REFERENCE LINE (0,0) (5,9) (10,18)

number of one-hour intervals with average rates of 1.8 (Circle one): 0 1 2 3 4 5

4pts (d) Suppose water flows into the reservoir at a constant rate of 1.8 thousand gallons per hour. What is the smallest amount of water needed in the reservoir at midnight so that the reservoir never has a shortage in this 12-hour period?

LARGEST GAP SIZE

RANGE: 8 TO 10

~9 thousand gallons

$\boxed{\text{V1}}$

3. (18 points) You sell Things.
You are given that the total cost for selling x hundred Things is $TC(x) = 12x + 4000$ hundred dollars.
Also, you are told that the selling price per Thing is $p = -3x + 600$ dollars/Thing where x is in hundred Things.

$\boxed{\text{2 pts}}$ (a) Give the formula for total revenue, $TR(x)$.

$TR(x) =$ $-3x^2+600x$

(b) Recall in this case that $MR(x) = \frac{TR(x+0.01)-TR(x)}{0.01}$. Find and completely simplify the formula for Marginal Revenue.

$\boxed{\text{5 pts}}$

+2 FOR START $\rightarrow$ $[-3(x+0.01)^2+600(x+0.01)] - [-3x^2+600x]$

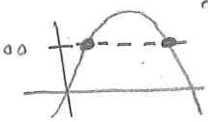
$= -3(\cancel{x^2}+0.02x+0.0001) + \cancel{600x} + 6 + \cancel{3x^2} - \cancel{600x}$

$+2 \rightarrow$ $= -0.06x - 0.0003 + 6 = -0.06x + 5.9997$

$+1 \rightarrow$ so $MR(x) = \dfrac{-0.06x+5.9997}{0.01} = -6x + 599.97$

$MR(x) =$ $-6x + 599.97$ dollars/Thing

$\boxed{\text{5 pts}}$ (c) Find the largest interval over which Total Revenue is greater than or equal to \$3000. (Round answers to two digits after the decimal)



$+1 \rightarrow$ $-3x^2+600x \stackrel{?}{=} 3000$

$+1 \rightarrow$ $-3x^2+600x-3000=0$

$+2 \rightarrow$ $x = \dfrac{-600 \pm \sqrt{600^2-4(-3)(-3000)}}{2(-3)} = \dfrac{-600 \pm \sqrt{324000}}{-6} = \dfrac{-600 \pm 569.2099788}{-6}$

$+1 \rightarrow$ $x \approx 5.13167$ or $x \approx 194.8683$

$x =$ 5.13 to $x =$ 194.87 hundred Things

$\boxed{\text{6 pts}}$ (d) Find the quantity and selling price which correspond to maximum profit.

TWO OPTIONS

+4 ① $MR = MC \Rightarrow -6x + 599.97 = 12 \Rightarrow x = 97.995$ hundred Things

② PROFIT $= -3x^2 + 588x - 4000 \Rightarrow x = -\dfrac{588}{2(-3)} = 98$ hundred Things

+2 "price at $x=98$" $= -3(98)+600 = 306$

BOTH ROUND TO THIS

NOTE:
$-3(97.995)+600$
≈ 306.02

Quantity: $x =$ 98 hundred Things

Selling Price: $p =$ 306 dollars/Thing

ACCEPT THIS

V1

4. (16 pts) The average variable cost of producing x **thousand** items is given by

$$AVC(x) = 0.01x^2 - 0.7x + 80 \quad \text{and} \quad MC(x) = 0.03x^2 - 1.4x + 80,$$

where $AVC(x)$ and $MC(x)$ are in dollars/item.
In addition, the selling price per item is a constant $p = 86$ dollars/item.

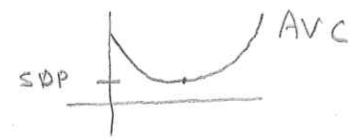
6pts (a) Give the formulas/values for all the following:

+2 i. Variable Cost: $VC(x) =$ $0.01x^3 - 0.7x^2 + 80x$ thousand dollars

+2 ii. Total Revenue: $TR(x) =$ $86x$ thousand dollars

+2 iii. Marginal Revenue: $MR(x) =$ 86 dollars/item

5pts (b) Recall that the Shutdown Price (SDP) is the lowest value of $AVC(x)$. Find the Shutdown Price. (Round to the nearest cent)



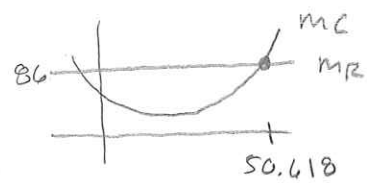
$$+3 \left\{ x = -\frac{-0.7}{2(0.01)} = 35 \right.$$

$$+2 \left\{ AVC(35) = 0.01(35)^2 - 0.7(35) + 80 = 67.75 \right.$$

SDP = 67.75 dollars/item

5pts (c) Find the quantity at which profit is maximized. (Round to three digits after the decimal).

MUST USE MR = MC APPROACH (OR USE CALCULUS)



$$+2 \left\{ \begin{array}{l} 86 \stackrel{?}{=} 0.03x^2 - 1.4x + 80 \\ 0 = 0.03x^2 - 1.4x - 6 \end{array} \right.$$

$$+3 \left\{ x = \frac{1.4 \pm \sqrt{1.4^2 - 4(0.03)(-6)}}{2(0.03)} = \frac{1.4 \pm \sqrt{2.68}}{0.06} = \frac{1.4 \pm 1.63707055}{0.06} \right.$$

$$+1 \left\{ x \approx -3.9511759 \quad \text{or} \quad x \approx 50.6178425 \right.$$

$x =$ 50.618 thousand items

V1

5. (16 pts) (For all your work below, round your **final answer** to two digits after the decimal)

5 pts (a) Jill found an investment that will pay her 5% annual interest, compounded quarterly. How much must Jill invest in the account now so that she will have \$10,000 in five years?

+1 $\{ \quad B = P\left(1+\frac{r}{m}\right)^{mt}$

+2 $\left\{ \begin{array}{l} \left.\begin{array}{l} r = 0.05 \\ m = 4 \\ t = 5 \end{array}\right\} \quad \begin{array}{l} \frac{r}{m} = 0.0125 \\ mt = 20 \end{array} \\ 10000 = P(1.0125)^{20} \end{array} \right.$

+2 $\left\{ \begin{array}{l} 10000 = P \cdot 1.282037232 \\ P \approx 7800.08548 \end{array} \right.$

7800.09 dollars

5 pts (b) Molly deposits \$500 into an account that pays 3% annually, compounded continuously. How long will it take for the account balance to double?

+1 $\{ \quad B = Pe^{0.03t}$

+2 $\left\{ \begin{array}{l} 1000 = 500\, e^{0.03t} \\ 2 = e^{0.03t} \\ \ln(2) = 0.03t \end{array} \right.$

+2 $\left\{ \quad t = \frac{\ln(2)}{0.03} \approx 23.104906 \right.$

23.10 years

6 pts (c) Fred has an account that pays interest compounded semi-annually. He deposited \$600 initially and then 5 years later the account balance was \$900. What is the interest rate?

+1 $\{ \quad B = P\left(1+\frac{r}{2}\right)^{2t}$

+1 $\{ \quad 900 = 600\left(1+\frac{r}{2}\right)^{10}$

+1 $\{ \quad 1.5 = \left(1+\frac{r}{2}\right)^{10}$

+2 $\left\{ \begin{array}{l} (1.5)^{1/10} = 1 + \frac{r}{2} \\ 1.041379744 = 1 + \frac{r}{2} \end{array} \right.$

$\left. \begin{array}{l} 0.041379744 = \frac{r}{2} \\ 0.082759488 = r \end{array} \right\}$ +1

8.28 %

6. (14 pts) (Round to the nearest cent)

7pts (a) You plan to take a big trip in four years (after college). You deposit \$100 at the beginning of every month for 4 years in an account with 6% annual interest compounded monthly. How much money will be in the account after 4 years AND how much interest do you earn?

$r = 0.06$, $m = 12$, $t = 4$ $\quad i = r/m = 0.005$, $n = mt = 48$ PAYMENTS

$$+1 \left\{ F = R\,\frac{(1+i)^n - 1}{i}\,(1+i) \right.$$

$$+2 \left\{ F = 100\left(\frac{(1.005)^{48} - 1}{0.005}\right)(1.005) \right.$$

$$+1 \left\{ = \$5436.83 \right.$$

−2 IF THEY USE ORDINARY ANNUITY

$$+3 \left\{ \begin{aligned} \text{INTEREST} &= 5436.83 - 48 \times 100 \\ &= 636.83 \end{aligned} \right.$$

Balance in 4 years = 5436.83 dollars

Total interest earned = 636.83 dollars

7pts (b) Your friend also plans to save for a trip in four years. They plan to make deposits at the **end** of every month for 4 years in an account with 6% annual interest compounded monthly. If your friend knows they will need \$15,000 for their trip how much money do they need to deposit at the end of every month?

−2 IF THEY USE ANNUITY DUE

$$+1 \left\{ F = R\,\frac{(1+i)^n - 1}{i} \right.$$

$$+3 \left\{ 15000 = R\,\frac{(1.005)^{48} - 1}{0.005} \right.$$

$$+3 \left\{ \begin{aligned} 15000 &= R \times 54.09783222 \\ R &= 277.2754 \end{aligned} \right.$$

277.28 dollars