1. (10 pts) You sell Things. The total revenue, $TR(x)$, and total cost, $TC(x)$, in dollars on an order of x Things are given by

$$TR(x) = 30x - 0.25x^2 \text{ dollars, and } TC(x) = 13x + 100 \text{ dollars.}$$

If rounding is necessary, round final answers to the nearest Thing or nearest cent.

(a) Find the formulas for Variable Cost and Average Cost.

$VC(x) =$ $13x$

$AC(x) =$ $\frac{13x+100}{x} = 13 + \frac{100}{x}$

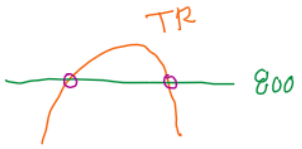
(b) Find the largest interval on which Total Revenue is great than or equal to 800 dollars.

$30x - 0.25x^2 \stackrel{?}{=} 800$

$\Rightarrow 0 = 0.25x^2 - 30x + 800$

$\Rightarrow x = \frac{30 \pm \sqrt{30^2 - 4(0.25)(800)}}{2(0.25)}$

$= \frac{30 \pm \sqrt{100}}{0.5} = \frac{30 \pm 10}{0.5}$ $\nearrow \frac{40}{0.5} = 80$ $\searrow \frac{20}{0.5} = 40$



$q =$ 40 to $q =$ 80 Things

(c) Find the maximum profit.

$P(x) = TR(x) - TC(x) = (30x - 0.25x^2) - (13x + 100)$

$\Rightarrow P(x) = -0.25x^2 + 17x - 100$

$x = \frac{-17}{2(-0.25)} = \frac{17}{0.5} = 34$

$P(34) = -0.25(34)^2 + 17(34) - 100$

$= 189$

189 dollars

(d) Recall: $MR(q) = TR(q+1) - TR(q)$. Find and completely simplify the formula for Marginal Revenue.

$[30(x+1) - 0.25(x+1)^2] - [30x - 0.25x^2]$

$30x + 30 - 0.25(x^2 + 2x + 1) - 30x + 0.25x^2$

$30x + 30 - 0.25x^2 - 0.5x - 0.25 - 30x + 0.25x^2$

$29.75 - 0.5x$

$MR(x) =$ $29.75 - 0.5x$