**Math 111 Winter 2008 Exam 1 Comments**

The first exam is often an eye opener in this course. The concepts are relatively simple, so students have a false sense that they don't need to study for the exam. The application of the concepts and the recognition of when to use them is what is difficult. Many of the questions were identical to problems from old exams. So if you missed them, then you either didn't study the old exams or you didn't understand them well enough when you did study them.

Now is the time to review your studying techniques so that you can improve for the second exam. To help you figure out what went right and what went wrong, I have explained each problem below along with where you should have seen it when you were studying. I hope that this gives you an idea of my perspective and helps you to understand how to change your study habits.

**THE EXPLANATIONS BELOW ARE FOR VERSION 1**. (For version 2 the numbers are changed but the explanations of how to solve the problems are the same.)

**The solutions for both versions are currently posted on the website. Please carefully read the explanations below and the solutions online before you ask your TA or me about the grading of your exam.**

## PROBLEM 1: Cost Analysis

This problem was related to WS 3, 8, and 9. The Total Cost (TC) graph is given, but you should immediately know what the Variable Cost (VC) graph looks like (it would be shifted down by the FC=125).
I thought that 1(a) through 1(d)(i) on this problem would be the easiest on the exam. Problem 1(d)(ii) should have been easy if you were attending class regularly (this type of terminology was used in lecture often).

1. **To find VC:** (very similar to Taggart Fall 2007 1(a)(iii) and Nichifor Fall 2006 1(a)) Recall $TC(q) = VC(q) + FC$. Since $FC = 125$ and $TC(600) = 350$, the equation says $350 = VC(600) + 125$. So

$$VC(600) = 350 - 125 = 225.$$

2. **To find MC:** (very similar to Nichifor Fall 2006 1(c), Fall 2007 5(a), and Fall 2004 1(a), and on WS 9/13(b)) To get a precise answer you need to use the fact that $MC(100)$ is the slope of the secant from 100 to 101 (which looks like a tangent line). You need to use two points to find the slope of the line.

3. **To find the BEP:** (very similar to Taggart Fall 2007 1(a)(iii), Loveless Fall 2005 2(b), Fall 2004 1(a), and WS 8/4,5,6) In lecture, whenever I gave the definition I underlined (often I doubled or tripled underlined) the word slope. BEP = "the slope of the lowest diagonal line to $TC$". In this case we were always talking about a diagonal line (that is through the origin).

   We did many of these problems in class and they appear on several old exams. If you didn't give the slope, then you lost 3 points (this is because the fact that slope is the price was very important and heavily emphasized in lecture).

   *If you missed this one, then either you just made a silly mistake or you didn't study! Over 20 percent of the class missed some or all of this problem. This means that many of you need to make significant changes to your studying habits.*

4. **Market Price = \$1.50, what is profit at $q = 300$?** Recall $P = TR - TC$. So you needed to find $TR$ at $q = 300$ and $TC$ at $q = 300$ and subtract the values. To help you visualize, you could have drawn the $TR$ line (with slope 1.5). You need to find the distance that $TR$ is above $TC$ at $q = 300$.

5. **Market Price = \$1.50, at $q = 150$ how much of the $FC$ is recovered?** This is something we talked about in lecture on the Friday, Monday and Wednesday before the test. In your notes, you will see a few spots where I labeled the graph "this is the amount of FC recovered". So if you were attending lecture and paying attention, then you should have been familiar with this terminology. (I intended this as a free-be for those that regularly attend lecture).

   Any value of $TR$ that is between $TC$ and $VC$ is a value at which profit is negative, but some of the FC is recovered. The distance below $TC$ is the (negative) profit, the distance above $VC$ is the amount of $FC$ recovered.

   So you need to start by either finding profit (and taking this away from FC) or finding $VC$ and subtracting this from $TR$.

## PROBLEM 2: Distance and Speed

I thought that 2(a)(b) and (d) would be relatively straightforward questions. 2(c) was slightly more difficult, but you should have been able to translate average speed.

1. **To find lowest ATS:** (very similar to Taggart Winter 2006 1(a), Loveless Fall 2005 1(b), and Conroy Fall 2005 3(b)) Find the *slope* of the lowest diagonal line to the Car A graph.
2. **To find a 2-hour interval when average speed = 30 mph:** (very similar to Loveless Fall 2005 1(d)) The 30 mph is a slope. So you need to draw a reference line with slope=30. Then slide, or roll, your ruler to a 2-hours interval with this slope.
3. **Find a one-hour interval over which Car A and Car B have the same average speed.** (similar to Taggart Fall 2006 1(c) and WS 1/13(e)) You are looking for a one-hour interval where the slope of the secant to Car A is the same as (parallel to) the slope of the secant to Car B.
4. **Translate** $\frac{B(4)}{4} < \frac{A(5)-A(3)}{2}$ (similar to Loveless Fall 2005 3(b), Taggart Winter 2006 3(c), Nichifor Fall 2007 2(a), and WS 7/10(g)) The left-side is the slope of the diagonal to the Car B graph at $t = 4$ and the right-side is the slope of the secant to the Car A graph from $t = 3$ to $t = 5$. The left side clearly has a steeper slope. (So it is false as written).

## PROBLEM 3: Reservoir

I thought that 3(a)(b) and (d) would be relatively straightforward questions. 3(c) was slightly more difficult, but you should have been able to translate that the problem was about slopes of tangents.

1. **Least amount of water needed:** (very similar to Winter 2007 3(d), Fall 2004 1(b), Winter 2004 1(c), WS 2/10(b)(c)(d), and WS 7/10(b)) Find the biggest gap where output ($O(t)$) is above input ($I(t)$).
2. **Find the longest time when average overall rate of flow into the reservoir is decreasing:** (very similar to Nichifor Fall 2007 5(c), Winter 2004 1(b), and identical to WS 5/14(c)) When you see "overall rate" you should immediately recognize that this is the talking about the slope of diagonal lines on $I(t)$. Fix your ruler at the origin and rotate. You are looking for the longest time when the ruler is getting less steep. This happens from $t = 0$ to about $t = 3.1$.
3. **Find when** $\frac{O(t+0.1)-O(t)}{0.1} > \frac{I(t+0.1)-I(t)}{0.1}$**.** (similar to Taggart Winter 2006 3(e), Fall 2004 1(e), WS 6/8(e)(i), and WS 7/10(g)) You should recognize that both sides are slopes of secants between points that are very close together (so they look like tangents). You are looking for times when the slope of tangent to $O(t)$ is steeper than the slope of the tangent to $I(t)$. The fast way to to this is to first look for when the slopes are the same (they are about the same at around $t = 1$, $t = 4.5$ and $t = 8.7$). Before $t = 1$ the $I(t)$ graph is steeper than the $O(t)$ graph. From $t = 1$ to $t = 4.5$, the slope of $O(t)$ is steeper than $I(t)$. From $t = 4.5$ to $t = 8.7$, the slope of $I(t)$ is steeper than $O(t)$. From $t = 8.7$ to $t = 10$ the slope of $O(t)$ is steeper than $I(t)$.
4. **Translate the incremental rate of flow out from $t$ to two-hours later is 2 thousand gallons.** (similar to Loveless Fall 2005 3(d) and WS 7/11(a)) If you practiced translations, then this is routine. Incremental rate of flow out $= \frac{O(b)-O(a)}{b-a}$. The start is $a = t$ and the end is $b = t + 2$ (two-hours later). So $\frac{O(t+2)-O(t)}{2} = 2$. You need to draw a reference line with slope=2 and then you need to slide or roll your ruler to find an interval of length 2 with this slope. S

**Aside about Slopes** Remember that SLOPE $= \frac{\text{RISE}}{\text{RUN}}$. That is, the $y$ values should go on top and the $x$ values should go on the bottom. A large number of you (over 15% of the class) incorrectly computed slope as Run over Rise at some point on the exam. Please be careful to avoid this mistake in the future.

**My Expectations when I made the exam:** If you completely missed the 2 more difficult parts (2(c) and 3(c)) and you randomly missed another problem, then you would have lost 12 points. However, even for the more difficult problems you should have been able to get partial credit. I expected that students who studied old exams and attended lecture regularly would lose fewer than 10 points out of 50. Everyone makes the occasional silly mistake, but, from my perspective, if you study effectively, then you can guarantee yourself a grade above 75% on the exams (and if you don't make silly mistakes an effective studier gets grades about 90%).

One tangible thing that you can do to improve is to start visiting my office, or your TA's office hours, every week. I want you to do well in this course, so please change your habits and come ask me and your TAs for help.