

## Math 111 Week 2 Review

This review is not all inclusive. You are expected to know how to do all the problems in the homework.

### Worksheet 3: Revenue and Cost

- $TR$  = Total Revenue = total money brought in from a sale of  $q$  items.  
 $TC$  = Total Cost = total cost of producing  $q$  items.  
 $P$  = Profit =  $TR - TC$ .
- MAX PROFIT METHOD 1:** The maximum profit occurs at the quantity where the  $TR$  graph has the biggest gap above the  $TC$  graph.
- $MR$  = Marginal Revenue = money brought in from selling an additional item  
 That is,  $MR = \Delta TR$  = the change in  $TR$  from  $q$  to  $q + 1$ .  
 $MC$  = Marginal Cost = cost of producing an additional item  
 That is,  $MC = \Delta TC$  = the change in  $TC$  from  $q$  to  $q + 1$ .  
 If  $MR > MC$  at  $q$ , then profit increases from  $q$  to  $q + 1$ .  
 If  $MR < MC$  at  $q$ , then profit decreases from  $q$  to  $q + 1$ .
- MAX PROFIT METHOD 2:** The maximum profit occurs at the first quantity when  $MR < MC$  (that is, at the quantity when it switch from  $MR > MC$  to  $MR < MC$ ).

### Worksheet 4: Connections between WS 1, 2, 3.

- ‘Delta’ notation:  $\Delta$  = ‘change in’ a value. For example:

  - $\Delta t$  = (end time) - (start time).
  - Since  $\Delta y = y_2 - y_1$  and  $\Delta x = x_2 - x_1$ , we see that ‘the slope of a line’ =  $\frac{\Delta y}{\Delta x}$ .
- When we fix  $\Delta t$ , or  $\Delta q$ , then we are considering increments.
- Here are the connections between the three scenarios from WS 1, 2, and 3:

	Car	Reservoir	Business
increment	$\Delta t = 5 \text{ min}$	$\Delta t = \frac{1}{2} \text{ hour}$	$\Delta q = 1$
change in value	$\Delta D$	$U = \Delta O$	$MR = \Delta TR, MC = \Delta TC$
incremental rate of change	$AS = \frac{\Delta D}{\Delta t}$	$\frac{\Delta O}{\Delta t}$	$MR = \frac{\Delta TR}{\Delta q}, MC = \frac{\Delta TC}{\Delta q}$
overall rate of change	$ATS = \frac{D}{t}$	$\frac{O}{t}$	$AR = \frac{TR}{q} = \text{price}, AC = \frac{TC}{q}$

	Graphical Interpretation
increment	change on the $x$ axis
change in value	change in height of the graph
incremental rate of change	slope of a secant line
overall rate of change	slope of a diagonal line

- How this is useful?
  - To compute  $MR$ ,  $MC$ , or  $AS$ , you can use the “secant line method”. That is, draw the secant line, find two points and compute the slope.
  - To compute  $AR$ ,  $AC$ , or  $ATS$ , you can use the same method with a diagonal line.
  - To find lowest or highest  $AR$ ,  $AC$ , or  $ATS$ , you can use “ruler trick 1”. That is, fix the ruler at the origin and rotate.
  - To find lowest or highest  $MR$ ,  $MC$ , or  $AS$  over a given increment you can use “ruler trick 2”. That is, checking increments consecutively put your ruler on the secant line and compare slopes.

- (e) If two points are very close together it is often difficult to be precise when finding the change in height. So it may be better to use the secant line method to compute the slope first. Then use the slope to find the change in height.

**Worksheet 5: The Lagging Car**

1. A big part of this course is understanding how to read and compare two graph simultaneously and how to use notation to represent what you are observing. This worksheet introduces you to these concepts.
2. Functional Notation Examples
  - (a)  $D(t)$  = the distance (in miles) traveled by time  $t$  (in minutes).  
 $D(5) = 30$  means that “the car has traveled 30 miles by the time  $t = 5$  minutes” The number inside the parantheses is  $t$  and the other side of the equation is the distance.
  - (b)  $D(10) - D(5) =$  “the change in distance from  $t = 5$  to  $t = 10$ .”
  - (c)  $D(30) > D(20) =$  “the distance traveled in 30 minutes is greater than the distance traveled in 20 minutes”.
  - (d)  $\frac{D(40)}{40} =$  “the overall rate of change in distance at  $t = 40$  (this is the ATS at 40)”.
  - (e)  $\frac{D(50)-D(30)}{50-30} =$  “the incremental rate of change from  $t = 30$  to  $t = 50$  (this is the AS from 30 to 50)”.