Math 111 Week 9 Review

This review is not all inclusive. You are expected to know how to do all the problems in the homework.

Worksheet 20: Partial Compounding Periods

1. Here we found out that the multiplicative, additive and CAF formulas work with fractions for input.
2. WARNING: Make sure that your input matches up with the number of times that the multiplier is being used!
3. EXAMPLES:
   (a) Money gains interest at a rate of 50% every 4 years. What is the value of \$5,000 after 6 months?

   - ANSWER: Use the CAF formula, $A(t) = P(1+r)^t$.
     Then $P = \$5,000$, $r = 0.5$ every 4 years, so
     $$A(t) = 5000(1.5)^t.$$
     This means that $t = 1$ corresponds to 4 years.
     6 months $= \frac{1}{2}$ year we need to figure out what value of t this would be.
     Since $t = 1$ is 4 years, $t = 0.5$ is 2 year, $t = 0.25$ is one year, so half of this gives the correct t. We can also get this by dividing $\frac{1}{2}$ by 4. So
     $$t = \frac{1}{8} = 0.125.$$
     Thus, $A(0.125) = 5000(1.5)^{0.125} = 5259.9475 = \$5259.95$.

   (b) In general, if you can get the units to match up, then all you have to do is divide to get t.

4. IMPORTANT NOTE:
   "$(r \times 100)\%$ compounded at the end of year" is different than
   "$(\frac{r}{n} \times 100)\%$ compounded n times a year".
   - For example, "6% a year" is different than "3% twice a year". (The second actual turns out to be bigger than 6% a year). This means that in the example above, it would have been incorrect to try to divide the rate.
   - This is because if interest is compounded more often, then the interest is allowed to build on itself.

Worksheet 21: Bank Interest

1. In this section, we deciphered the information that is given by a bank.
2. SIMPLE INTEREST: An account that earns interest on the *principal only* is a **simple interest** account.
   If P=Principal, r=decimal interest rate, and t=number of years, then
   $$A(t) = P(1+rt) \qquad \text{(simple interest formula)}.$$
3. COMPOUNDING: An account that earns interest on the *entire balance* is a **compounding** account.
   (a) In t years, an account the pays a decimal interest rate r per year, compounded n times a year with principal P has balance given by
   $$A(t) = P\left(1 + \frac{r}{n}\right)^{nt} \qquad \text{(bank compounding formula)}.$$
   (b) In t years, an account the pays a decimal interest rate r per year, compounded **continuously** with principal P has balance given by
   $$A(t) = Pe^{rt} \qquad \text{(bank continuous compounding formula)}.$$
4. NOTE: For all of these *bank* formulas, t **is always in years**.

Worksheet 22: APY

1. In this section we learn how to compare bank accounts by finding what the equivalent yearly rate is for each account.
2. **Annual Percentage Yield (APY)** is the percentage of the balance that the account earns in interest each year. We can always compute it using the percentage change formula over one year

$$\text{APY } = \frac{\text{NEW - OLD}}{\text{OLD}} \times 100\% = \frac{A(1) - A(0)}{A(0)} \times 100\%.$$

3. For the compounding formula this simplies as follows:
   (a) For compounding n times a year,

$$\text{APY } = \left[\left(1 + \frac{r}{n}\right)^n - 1\right] \times 100\%.$$

   (b) For continuous compounding,

$$\text{APY } = [e^r - 1] \times 100\%.$$