Math 111 Refresher for Math 112 Students

1. **Rates of Change for some function $f(t)$**

- $\frac{f(b)-f(a)}{b-a}$ = "Rate of Change from $t = a$ to $t = b$"
  = "the slope of the secant line through the graph at $t = a$ and $t = b$".
  - If $a = 0$, then we call this the overall rate of change. Otherwise we call it the average rate of change over an interval.
- The equation $\frac{f(t)}{t}$ = "The slope of the diagonal line through the graph of $f(t)$".

2. **Business Terms and Techniques**

- *Basic Functions Relating to Business*: For these functions, you plug in a quantity, q, and the function outputs a dollar amount which is related to the amount of money in, the amount of money out, or both.
  - $TR = TR(q) = pq$ = total revenue for selling q Things (money brought in before cost)
  - $TC = TC(q) = VC(q) + FC$ = total cost for selling q Things.
    - $FC = TC(0)$ = fixed cost = 'y'-intercept of the TC graph.
    - $VC = VC(q)$ = variable cost = cost not including the fixed cost.
  - $P = P(q)$ = Profit = $TR(q) - TC(q)$.
- *Average Business Functions*: For these functions, you plug in a quantity, q, and the function outputs the dollars per item.
  - $AR = AR(q) = \frac{TR(q)}{q}$ = average revenue = price per item when selling q items.
  - $AC = AC(q) = \frac{TC(q)}{q}$ = average cost = cost per item when producing q items.
  - $AVC = AVC(q) = \frac{AVC(q)}{q}$ = average variable cost = variable cost per item when producing q items.
- *Marginal Functions*: For these functions, you plug in a quantity, q, and the function outputs the dollar amount that comes about when the quantity increases by 'one unit'.
  - $MR = MR(q) = TR(q + \text{'one unit'}) - TR(q)$
    = "change in revenue when selling one more item after q"
    = "slope of the secant line through the TR graph from q to $q+$'one unit'."
  - $MC = MC(q) = TC(q + \text{'one unit'}) - TC(q)$
    = "change in cost when selling one more item after q"
    = "slope of the secant line through the TC graph from q to $q+$'one unit'."
- *Maximum Profit*: We have many ways to find the maximum profit. Here are a few:
  - *Graphically*
    - Find the quantity where the TR graph is above the TC graph by the greatest difference.
    - Find the quantity where the slope of the TR graph is the same as the slope of the TC graph.
  - *Symbolically*
    - Solve for q in the equation $MR(q) = MC(q)$.
    - If $P(q) = TR(q) - TC(q)$ is a quadratic function, then we can use the vertex formula.

3. **Quadratic Functions**

- A quadratic function is any expression of the form $ax^2 + bx + c$ with $a \neq 0$.
  - If $a < 0$, then the parabola opens downward (it is frowning).
  - If $a > 0$, then the parabola opens upward (it is smiling).
- To find x coordinate of a vertex, we use the **vertex formula**: $x = -\frac{b}{2a}$. To get the maximum/minimum value of the function, we then plug this value back into the function. The vertex is a maximum if the parabola opens downward and a minimum if the parabola opens upward.
- To solve an equation involving a quadratic, get everything to one side of the equation so that it looks like: $ax^2 + bx + c = 0$ and use the **quadratic formula** (if you are entering it into your calculator remember to put parentheses where I have indicated):

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{\left(-b \pm \sqrt{(b^2 - 4ac)}\right)}{(2a)} .$$