

MATH 112
Final Exam
Spring 2018

Name _____

Student ID # _____

Section _____

HONOR STATEMENT

“I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam.”

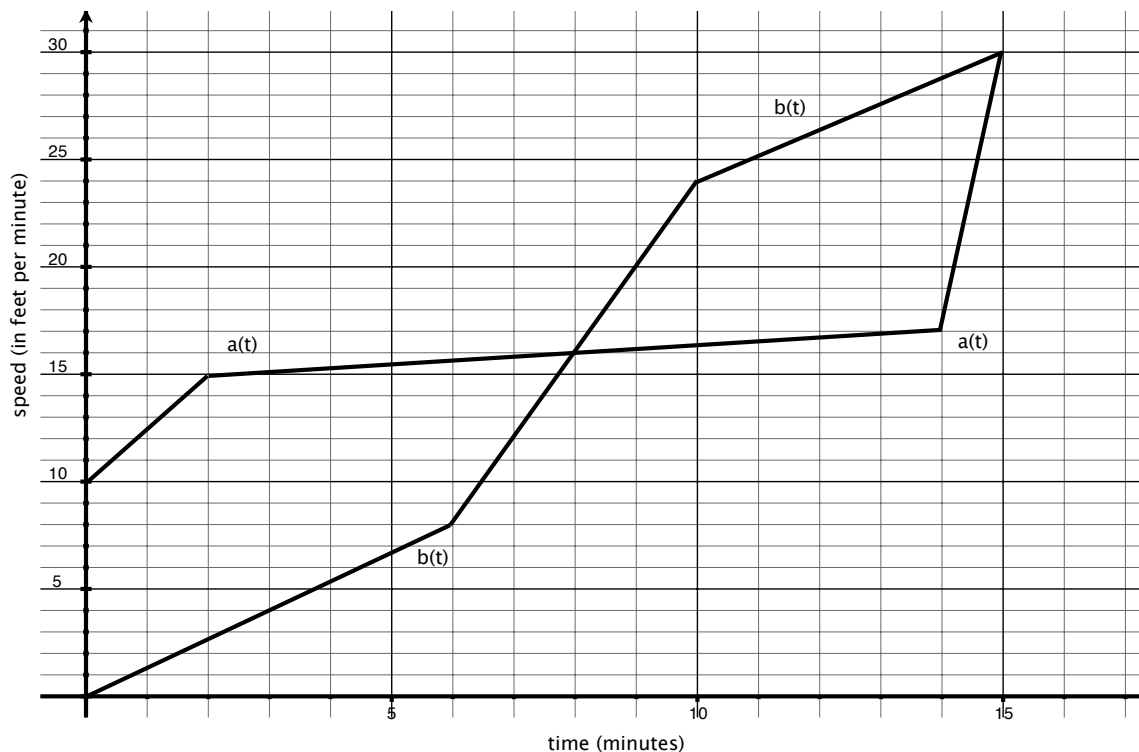
SIGNATURE: _____

1	28	
2	16	
3	15	
4	17	
5	12	
6	12	
Total	100	

- Your exam should consist of this cover sheet, followed by 6 problems on 7 pages. Check that you have a complete exam.
- You are allowed to use a TI30-XIIS calculator, a ruler, and one sheet of hand-written notes. All other sources are forbidden.
- Do not use scratch paper. If you need more room, use the back of the page and indicate to the grader you have done so.
- Turn your cell phone OFF and put it away for the duration of the exam.
- You may not listen to headphones or earbuds during the exam.
- Unless otherwise indicated, you must use the methods of this course and show all of your work. Clearly label lines and points that you are using and show all calculations. The correct answer with little or no supporting work may result in no credit. If you use a guess-and-check method when an algebraic method is available, you may not receive full credit.
- Unless otherwise indicated, you may round your **final answer** to two digits after the decimal.
- There are multiple versions of the exam, you have signed an honor statement, and cheating is a hassle for everyone involved. DO NOT CHEAT.

GOOD LUCK!

1. (28 points)



The graph above shows the instantaneous speed of two cars, Car A and Car B. The two cars are next to one another on the road at $t = 0$. We denote the relevant functions as follows:

$$A(t) = \text{distance covered by Car A by time } t \quad a(t) = \text{speed of Car A at time } t$$

$$B(t) = \text{distance covered by Car B by time } t \quad b(t) = \text{speed of Car B at time } t$$

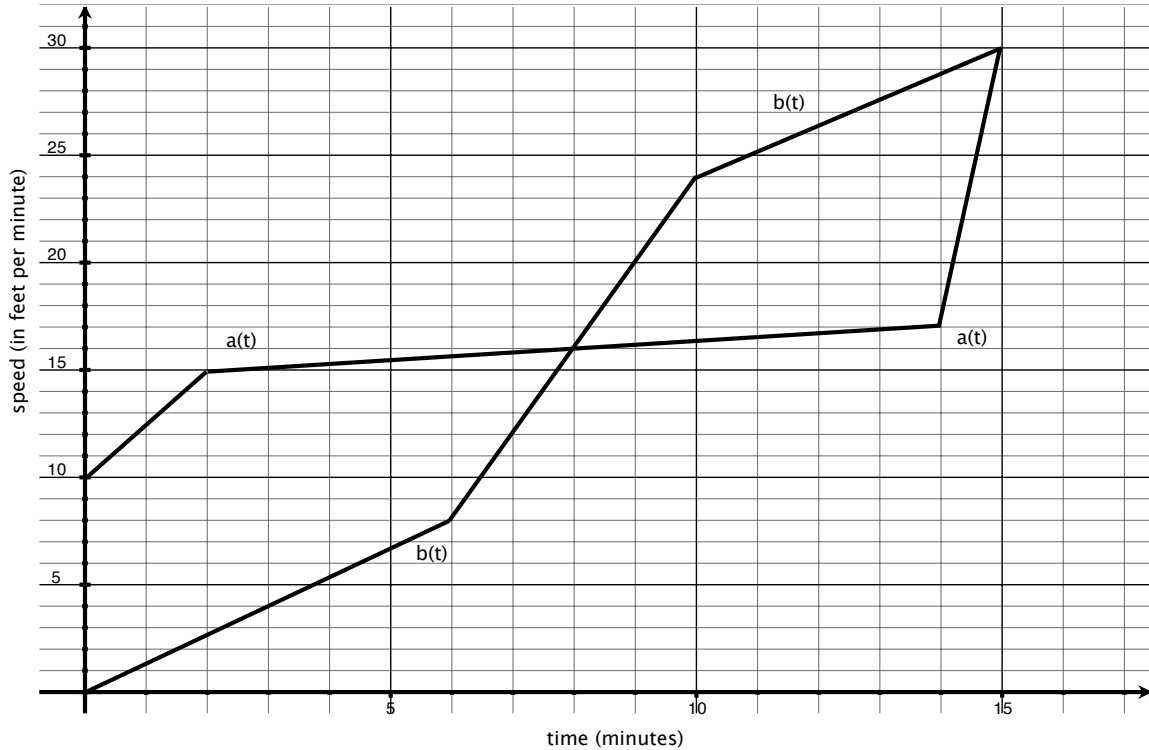
$$D(t) = A(t) - B(t)$$

(a) Determine whether each of the following is true (**T**) or false (**F**). You do not need to show any work. If I can't read your answer, you get no credit.

- i. _____ $a(t) = A'(t)$
- ii. _____ $A(t) = a'(t)$
- iii. _____ The distance between the cars at $t = 5$ is $a(5) - b(5)$.
- iv. _____ The cars are farthest apart at $t = 8$.
- v. _____ The cars are in the same place at $t = 8$.
- vi. _____ $A'(2) > B'(2)$
- vii. _____ $A(8) > B(8)$
- viii. _____ $B(9) > A(9)$
- ix. _____ $D(0) = 10$
- x. _____ $D(10) = \int_0^{10} a(t)dt - \int_0^{10} b(t)dt$

THIS PROBLEM IS CONTINUED ON THE NEXT PAGE.

Here are those graphs again:



- (b) Give the distance between the cars at $t = 2$.

ANSWER: _____ feet

- (c) Give a one-minute interval during which $D(t)$ increases and then decreases.

ANSWER: from $t =$ _____ to $t =$ _____

- (d) Give the average speed of Car B during the first six minutes.

ANSWER: _____ feet per minute

- (e) How far does Car A travel from $t = 2$ to $t = 14$?

ANSWER: _____ feet

2. (16 points) You sell Items. Your marginal revenue and marginal cost (both in dollars per Item) are given by the formulas

$$MR(q) = 236.52 - 5q \text{ and } MC(q) = 3q^2 - 48q + 197,$$

where q is in **thousands of Items**.

- (a) Find the quantity at which TR changes from increasing to decreasing.

ANSWER: $q =$ _____ thousand Items

- (b) Find the quantity that maximizes profit.

ANSWER: $q =$ _____ thousand Items

- (c) Write out the formulas for total revenue and variable cost.

ANSWER: $TR(q) =$ _____

ANSWER: $VC(q) =$ _____

- (d) If you sell $q = 8$ thousand Items, then your profit is 38 thousand dollars. Find the value of your fixed costs.

ANSWER: $FC =$ _____ thousand dollars

3. (15 points) Compute the indicated derivative. You do not need to simplify your answers.

(a) $w = \left(\frac{7u - 6}{3u + 4}\right)^{-2}$

$$\frac{dw}{du} =$$

(b) $M(s, t) = s^2t^6 + \ln(10s^2 + 3t^9)$

$$M_s(s, t) =$$

$$M_t(s, t) =$$

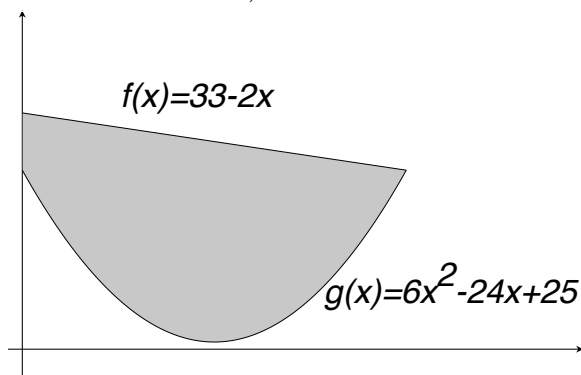
(c) $z = ye^{xy}$

$$\frac{\partial z}{\partial x} =$$

$$\frac{\partial z}{\partial y} =$$

4. (17 points)

- (a) Compute the area of the shaded region. (HINT: You will need to find where the two functions intersect.)



ANSWER: _____

- (b) The demand function for a product is $p = q^2 - 121.5q + 3780$, where p is in dollars per unit and q is in number of units. Find consumer surplus if equilibrium occurs at $q = 32$ units. **Round your final answer to the nearest dollar.**

ANSWER: _____ dollars

5. (12 points) You sell Things. The total revenue (in **hundreds of dollars**) for selling q **hundred Things** is given by the function $TR(q)$. You do not have the formula for $TR(q)$, but you know that, for any value of h ,

$$TR(q+h) - TR(q) = -6.8qh - 3.4h^2 + 40h.$$

- (a) Compute the change in TR that results if you increase production from 200 to 204 Things (i.e., from 2 to 2.04 hundred Things).

ANSWER: _____ hundred dollars

- (b) Compute the value of $\frac{TR(4.01) - TR(4)}{0.01}$.

ANSWER: $\frac{TR(4.01) - TR(4)}{0.01} =$ _____

- (c) Find the formula for $MR(q)$, the derivative of $TR(q)$.

ANSWER: $MR(q) = TR'(q) =$ _____

- (d) Your total cost is given by $TC(q) = 9.4q + 5$, where TC is measured in **hundreds of dollars** and q is in **hundreds of Things**. Find the quantity that maximizes profit.

ANSWER: $q =$ _____ hundred Things

6. (12 points) Suppose $f(x, y) = -5x^2 - 5x - 3y^2 + 4y + 10xy + 18$.

- (a) Find all pairs (x, y) which are candidates for a local maximum or a local minimum of $f(x, y)$. Show all work.

ANSWER: _____

- (b) Use partial derivatives to determine which of these numbers is bigger: A or B? Show all work.

$$A = \frac{f(2, 3.0001) - f(2, 3)}{0.0001} \quad B = \frac{f(2.0001, 3) - f(2, 3)}{0.0001}.$$

ANSWER: (circle one) A B is bigger

- (c) Consider the functions $f(2, y)$, $f(5, y)$, $f(8, y)$, and $f(10, y)$. Which has the steepest graph at $y = 1$? Show all work.

ANSWER: The function with the steepest graph at $y = 1$ is: _____