1. (10 points) The Cobb-Douglas production function for a certain Factory is given by the formula:

$$P = 1.7L^{0.3}K^{0.8}$$

where $P$ represents the factory's production (in hundreds of items), $L$ is the labor force (measured in workers) and $K$ is the capital investment (measured in **thousands of dollars**).

a) (2 pts) What is the production for a labor force of 100 workers and a capital investment of $2,500,000?

ANSWER: _____ hundred items

a) (4 pts) Compute the partial derivatives. Simplify your answers.

$$\frac{\partial P}{\partial L} =$$

$$\frac{\partial P}{\partial K} =$$

b) (4 pts) Suppose this factory has a labor force of 100 workers, and $1,500,000 in capital investment. Use a partial derivative to estimate the change in production if the capital investment stays the same, but an additional worker is hired. Show all steps.

ANSWER: _____ hundred items

Winter 2011

2. $Q(r,s) = \left(\frac{9s}{r}\right)^3 [r\ln(s)]^4$

$Q_s(r,s) =$

(b) (6 points) Let $f(x,y) = 40 + xy + \frac{1}{x} + \frac{125}{y}$. Find all points $(x,y)$ at which $f(x,y)$ may have a local optimum.

ANSWER: (list all pairs) $(x,y) =$_____

3. (13 pts) Let $z = f(x, y) = 14x - 12y + 3x^2y$.

   (a) (2 pts) Write out the formulas for $f_x(x, y)$ and $f_y(x, y)$.

   $f_x(x, y) =$ _____ $f_y(x, y) =$ _____

   (b) (4 pts) Find **all** points $(x, y)$ which are candidates for local maxima or local minima.

   ANSWERS: $(x, y) =$ _____

   (c) (3 pts) Suppose $(x, y) = (4, 0)$. circle the correct answer to complete the statement:
   A small increase in $x$ (with $y$ held fixed) leads to a (LARGER SMALLER EQUAL) increase in $z$ than a small increase in $y$ (with $x$ held fixed).
   Show appropriate calculations.

   ANSWER: (circle one) LARGER SMALLER EQUAL

   (d) (4 pts) If $y = -\frac{1}{3}$ is fixed, the function $g(x) = f(x, -\frac{1}{3})$ is a one variable function of $x$. By showing appropriate calculations, answer the following questions:

   i. Is $g(x)$ increasing, decreasing, or neither at $x = 3$?

   ANSWER: (circle one) INCREASING DECREASING NEITHER

   ii. Is $g(x)$ concave up, concave down, or neither at $x = 3$?

   ANSWER: (circle one) CONCAVE UP CONCAVE DOWN NEITHER

4. (8 points)

a) Suppose $g(x,y) = 3x^2 - 5x + 2x^2y - xy^2 + y^3 + 7$. Compute the following partial derivative:

$$g_x(x,y) =$$

b) Suppose

$$z = \frac{3y}{x^2+1} - xe^y + 2y\ln y.$$

Compute the following partial derivative:

$$\frac{\partial z}{\partial y} =$$

5. (5 points) You do not know the formula for a certain multi-variable function $f(x,y)$, but you are told that its two partial derivatives are:

$$f_x(x,y) = 2xy + 2y - 5$$

$$f_y(x,y) = x^2 + y - 4$$

Compute or approximate each of the following three values.

$$A = \frac{f(1,3.0001) - f(1,3)}{0.0001}$$

$$B = \frac{f(2.001,3) - f(2,3)}{0.001}$$

$C =$ the slope of the tangent line to the graph of $h(x) = f(x,2)$ at $x = 5$

ANSWER: A ≅_____, B ≅ _____, C = _____

**6.** Suppose $P(x, y) = x^3y + xy^2 - 3xy + 4x$.
Which graph is steeper: (A) $P(2, y)$ at $y = 5$; or (B) $P(x, 5)$ at $x = 2$?

ANSWER: circle one (A) (B)

Spring 2008

**7.** Let $f(x,y) = x^4y^3 - 3xy^2 + 4x^5 - \frac{6}{y^2} + (e^{x^3-x})(\ln y)$. Consider the three functions $f(1,y)$, $f(0,y)$, and $f(-1,y)$. Use a partial derivative to determine which of these functions has the steepest graph at $y = 1$.

ANSWER: (circle one) $f(1,y)$ $f(0,y)$ $f(-1,y)$
has the steepest graph at $y = 1$

8. (14 points) Consider the function $z = f(x, y) = -x^3 + 12xy - 4y^2 + 3y + 15$.

a) (4pts) Write out the two partial derivatives, $f_x(x, y)$ and $f_y(x, y)$. You need not show work.

$f_x(x, y) =$_____

$f_y(x, y) =$_____

b) (5 pts) Find the largest value of the function $f(x, 1)$ over the interval from $x = 0$ to $x = 3$.
Show all steps.

Answer: _____

d) (5 pts) Which graph is steeper:

i. the graph of the function $f(3, y)$ at $y = 4$ OR

ii. the graph of the function $f(x, 1)$ at $x = 2$ ?

Show all work.

Answer: _____ is steeper