

MATH 112
Final Exam
Spring 2016

Name _____

Student ID # _____

Section _____

HONOR STATEMENT

“I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam.”

SIGNATURE: _____

1	16	
2	19	
3	17	
4	16	
5	12	
6	20	
Total	100	

- Your exam should consist of this cover sheet, followed by 6 problems on 6 pages. Check that you have a complete exam.
- You are allowed to use a TI30-XIIS calculator, a ruler, and one sheet of hand-written notes. All other sources are forbidden.
- Do not use scratch paper. If you need more room, use the back of the page and indicate to the grader you have done so.
- Turn your cell phone OFF and put it away for the duration of the exam.
- You may not listen to headphones or earbuds during the exam.
- Unless otherwise indicated, you must use the methods of this course and show all of your work. Clearly label lines and points that you are using and show all calculations. The correct answer with little or no supporting work may result in no credit. If you use a guess-and-check method when an algebraic method is available, you may not receive full credit.
- Unless otherwise indicated, you may round your **final answer** to two digits after the decimal.
- There are multiple versions of the exam, you have signed an honor statement, and cheating is a hassle for everyone involved. DO NOT CHEAT.

GOOD LUCK!

1. (16 points) Two balloons are at the same altitude at $t = 0$. You are given the following information about them. t is measured in minutes and altitude is measured in feet.

Altitude vs. time: $A(t) = \frac{1}{3}t^3 - 5t^2 + 24t + 25$

for Balloon A

Rate of ascent: $A'(t) =$

for Balloon A

Altitude vs. time: $B(t) =$

for Balloon B

Rate of ascent: $B'(t) = 15 - 1.25t$

for Balloon B

- (a) Find the earliest time at which the balloons are rising at the same rate.

ANSWER: $t =$ _____ minutes

- (b) Find all times at which Balloon A changes from falling to rising.

ANSWER: (list all) $t =$ _____ minutes

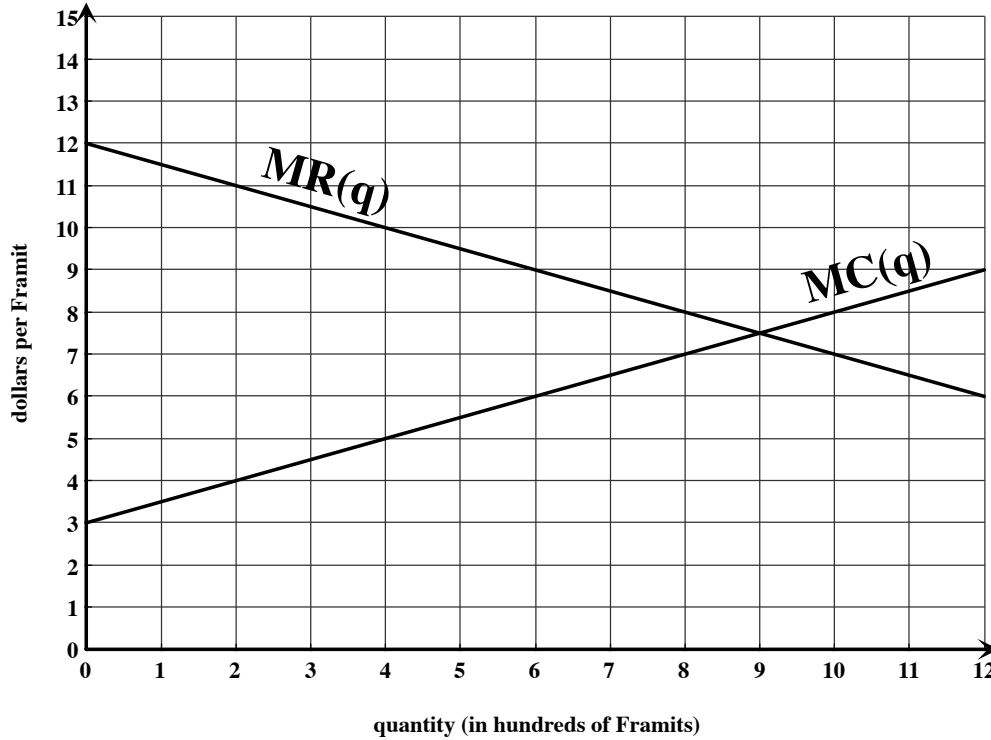
- (c) Use the fact that $A(0) = B(0)$ to find the formula for $B(t)$ without any undetermined constants.

ANSWER: $B(t) =$ _____

- (d) Find the highest altitude Balloon B reaches.

ANSWER: _____ feet

2. (19 points) The graph below shows marginal revenue and marginal cost for producing Framits.



(a) Compute the change in **total revenue** if quantity changes from 400 to 600 Framits.

ANSWER: _____ hundred dollars

(b) Fixed costs are 15 hundred dollars. Compute the **total cost** to produce 500 Framits.

ANSWER: _____ hundred dollars

(c) Again, fixed costs are 15 hundred dollars. Compute the maximum possible profit.

ANSWER: _____ hundred dollars

(d) If quantity changes from 200 to 201 Framits, will profit increase or decrease? By how much?

ANSWER:

Profit will (circle one) INCREASE DECREASE by _____ dollars.

(e) Compute the value of $TR''(3)$.

ANSWER: $TR''(3) =$ _____

3. (17 points) You produce two products: X -tel mobile phones and Y -pad tablet computers.
- (a) Your joint **total cost** (in **hundreds** of dollars) to produce x **hundred** phones and y **hundred** tablets is given by

$$C(x, y) = 4x^2 + 5xy + 16y + 50.$$

If you are currently producing 2 hundred phones and 1 hundred tablets, which will lead to a larger increase in **total cost**: producing one more phone or one more tablet?

Use partial derivatives to determine your answer and, as always, show all work.

ANSWER: producing one more (circle one) phone tablet

- (b) Your joint **total revenue** (in **hundreds** of dollars) to produce x **hundred** phones and y **hundred** tablets is given by

$$R(x, y) = -2x^2 - 2.7y^2 + 4xy + 5x + 2y + 100.$$

You know from experience that this revenue function has one critical point and that total revenue is maximized at that critical point. Find the maximum possible **total revenue**.

ANSWER: _____ hundred dollars

4. (16 points)

The supply function for a product is given by $p = 0.1e^{x/20}$, where x is the number of units and p is in dollars per unit.

The demand function is given by $p = 112.41 - 0.2x$.

The equilibrium price is \$85.41 per unit.

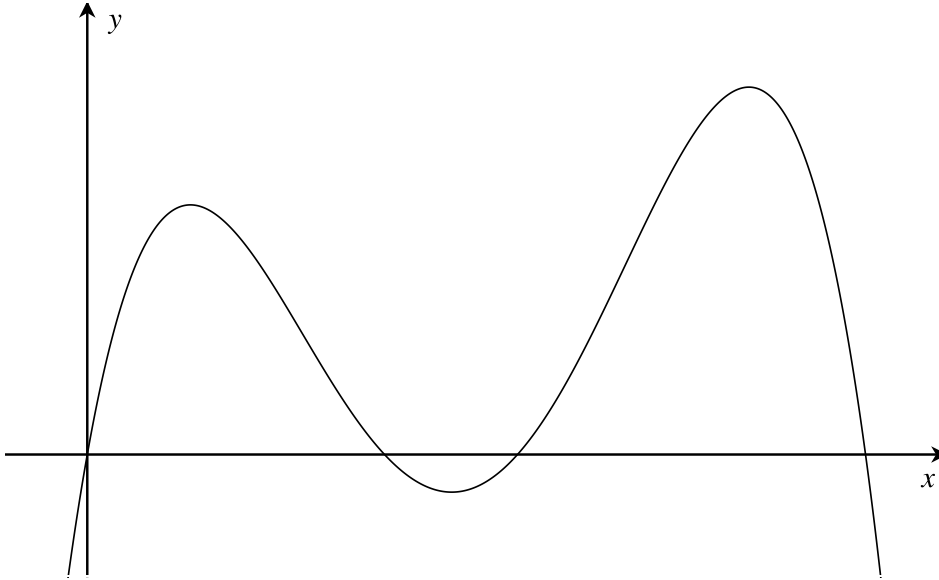
Find the consumer's surplus and producer's surplus at equilibrium.

(Round your answers to the nearest cent.)

ANSWER: consumer's surplus: \$ _____

producer's surplus: \$ _____

5. (12 points) The graph of the function $f(x) = -0.25x^4 + 3.5x^3 - 15x^2 + 20x$ is given below.



- (a) Find the equation $y = mx + b$ of the line tangent to $f(x)$ at $x = 0$.

ANSWER: $y =$ _____

- (b) Find the points of inflection. (There are two.)

ANSWER: $(x, y) =$ _____

$(x, y) =$ _____

- (c) Find the longest interval on which the graph of $f(x)$ is **concave up**.

ANSWER: from $x =$ _____ to $x =$ _____

6. (20 points)

(a) Compute the derivative of $y = \sqrt{9x^2 - \frac{1}{x} + \frac{\ln x}{6}}$.

Do not simplify. Put a box around your answer.

(b) Compute $R_x(x, y)$ if $R(x, y) = e^{(x^2y+4x)} + \frac{y \ln(x)}{y-x}$.

Do not simplify. Put a box around your answer.

(c) Compute $\int \frac{x^7 - 5}{x^4} + (x + 1) \left(\frac{1}{x} - 1 \right) dx$.

Put a box around your answer.

(d) Compute $\int_1^8 \frac{7}{2\sqrt[3]{x}} dx$.

Put a box around your answer.