MATH 112
Final Exam
March 10, 2007

Name SOLUTION OUTLINES

Student ID #________________ Section ________________

HONOR STATEMENT

"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

SIGNATURE:____________________________

1	14		6	10	
2	12		7	10	
3	10		8	12	
4	10		9	10	
5	12		Total	100	

- Your exam should consist of 9 problems. Check that you have a complete exam.
- Turn your cell phone OFF and put it away for the duration of the exam.
- Unless otherwise indicated, you must show your work. The correct answer with no supporting work may result in no credit.
- Unless otherwise indicated, you may round your FINAL ANSWER to two digits after the decimal.
- You may use a calculator for arithmetic purposes only (such as plugging into the quadratic formula or plugging into a function). ALL other work must be written and demonstrated on your exam. No credit will be given for guess-and-check or calculator methods, even if they give the correct answer.
- There are multiple versions of the exam. Any student found engaging in academic misconduct will receive a score of 0 on this exam.

GOOD LUCK!

1. (14 points) The total revenue and total cost for selling Frumtops are:

$$TR(q) = -5q^2 + 80q \text{ and } TC(q) = q^3 - 12q^2 + 60q + 60,$$

where q is in thousands of Frumtops and TR and TC are both in thousands of dollars.

(a) Use the derivative rules to find formulas for marginal revenue and marginal cost.

DERIVATIVES

ANSWER: $MR(q) =$____________________

$MC(q) =$____________________

(b) What does it cost to produce the $6{,}001^{st}$ Frumtop?

ASKING ABOUT MC AT 6000
WHICH IS $q = 6$ thousand Frumtops

COMPUTE MC(6)

ANSWER: ________________dollars

(c) What quantity maximizes profit?

SOLVE MR(q) = MC(q)

ANSWER: $q =$________________thousand Frumtops

(d) Find all quantities at which the slope of the tangent line to the marginal revenue graph is the same as the slope of the tangent line to the marginal cost graph.

SAME AS MR'(x)

SAME AS MC'(x)

FIND MR'(x) AND MC'(x)
SOLVE MR'(x) = MC'(x)

ANSWER: $q =$________________thousand Frumtops

(e) Is the graph of **total revenue** concave up, concave down, or neither at $q = 5$? (As always, you must show your work.)

NEED TO FIND TR''(x)
THEN COMPUTE TR''(5) IS IT POSITIVE OR NEGATIVE

ANSWER: (circle one) concave up concave down neither

2. (12 points) Let $y = f(x) = x^2 - 5x + 2\ln(x) + 6$

(a) Find all the critical numbers of $f(x)$.

FIND $f'(x)$
SOLVE $f'(x) = 0$

NOTE: $f'(x) = 2x - 5 + 2\frac{1}{x} \stackrel{?}{=} 0$
SO MULTIPLY BOTH SIDES BY x
TO GET $2x^2 - 5x + 2 = 0$

ANSWER: $x =$ ______________

(b) Use the Second Derivative Test to determine whether each critical number you found in part (a) gives a local minimum or a local maximum, or state that the test is inconclusive.

FIND $f''(x)$
PLUG EACH CRITICAL NUMBER FROM (a)
INTO $f''(x)$.

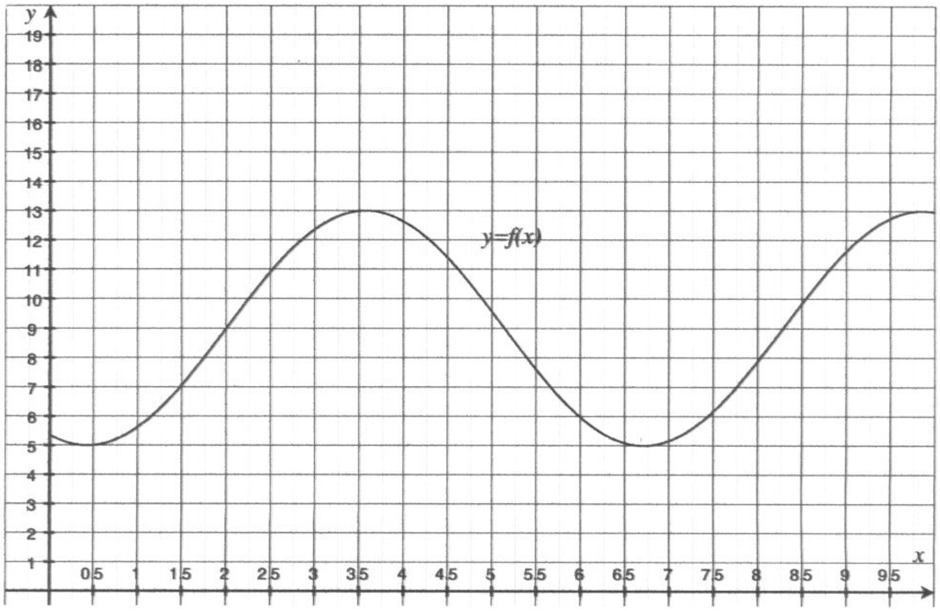
2nd Deriv. TEST →
IF f'' IS POSITIVE, THEN THE CRITICAL PT GIVES A LOCAL MIN.
IF f'' IS NEGATIVE, THEN THE CRITICAL PT GIVES A LOCAL MAX.

(c) Find the the global maximum and global minimum values of $f(x)$ on the interval from $x = 1$ to $x = 10$.

COMPUTE $f(1) = ?$, $f(10) = ?$
AND COMPUTE THE VALUE OF $f(x)$ AT EACH CRITICAL NUMBER BETWEEN 1 AND 10 FROM PART (a). BIGGEST OUTPUT = GLOBAL MAX
SMALLEST OUTPUT = GLOBAL MIN

ANSWERS: GLOBAL MINIMUM: ______________
GLOBAL MAXIMUM: ______________

4. (10 points) The graph below is of the function $y = f(x)$.



(a) Give two values of x at which $f'(x) = 2$.

WANT TO FIND WHEN THE SLOPE OF THE TANGENT LINE IS 2.
DRAW A LINE OF SLOPE 2. THEN FIND A TANGENT LINE WITH THAT SLOPE

ANSWER: $x =$ _______________ and $x =$ _______________

(b) Approximate the value of $\dfrac{f(4+h) - f(4)}{h}$ if $h = 0.001$.

$\dfrac{f(4.001) - f(4)}{4} \approx$ slope of the tangent at 4. DRAW IT. FIND THE SLOPE

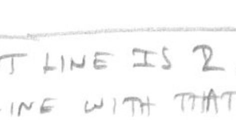
ANSWER: $\dfrac{f(4+h) - f(4)}{h} =$ _______________

(c) Give an interval of length 1 over which the graph of $f'(x)$ is positive and decreasing.

$f'(x)$ positive is the same as $f(x)$ increasing (0.5 TO 3.5 and 6.5 TO 10)
$f'(x)$ decreasing is the same as $f''(x)$ negative so $f(x)$ concave down (2 TO 5 AND 8 TO 10)

ANSWER: from $x =$ _______________ to $x =$ _______________

(d) Give an interval over which the graph of $f'(x)$ is shaped like this:



$f'(x) = 0 \iff f(x)$ horizontal tangent
$f'(x)$ negative $\iff f(x)$ decreasing
WE NEED HORIZ TANGENT, DEC, HORIZ. TANGENT

GIVE ANY ONE MINUTE INTERVAL WHERE BOTH ARE HAPPENING

ANSWER: from $x =$ _______________ to $x =$ _______________

5. (12 points) Two balloons are next to each other at time $t = 0$. You are given the following information about them:

Altitude vs. time: for balloon A	$A(t) = \frac{1}{3}t^3 - \frac{7}{2}t^2 + 10t + 50$
Rate of ascent: for balloon A	$A'(t) =$
Altitude vs. time for balloon B:	$B(t) =$
Rate of ascent for balloon B:	$B'(t) = 20 - 10t$

DERIVATIVE

ANTIDERIVATIVE

NOTE THAT $B(0) = A(0) = 50$ SO YOU CAN FIND C

where t is in minutes and Altitude is in feet.

(a) Find the highest altitude Balloon A reaches in the first 3 minutes.

GLOBAL MAX OF A FROM 0 TO 3
COMPUTE $A(0) = ?$ AND $A(3) = ?$
FIND CRITICAL NUMBERS $A'(t) \overset{?}{=} 0$ ← SOLVE
IF ANY CRITICAL # IS BETWEEN 0 AND 3
THEN COMPUTE THE VALUE OF $A(t)$ AT THAT VALUE
LARGEST OUTPUT

ANSWER: highest altitude = ________________ feet

(b) Use the fact that $B(0) = A(0)$ to determine the formula for $B(t)$ without any undetermined constants.

$B(t) = \int 20 - 10t \, dt = 20t - 5t^2 + C$
USE THE FACT THAT $B(0) = 50$ TO FIND C

ANSWER: $B(t) =$ ________________

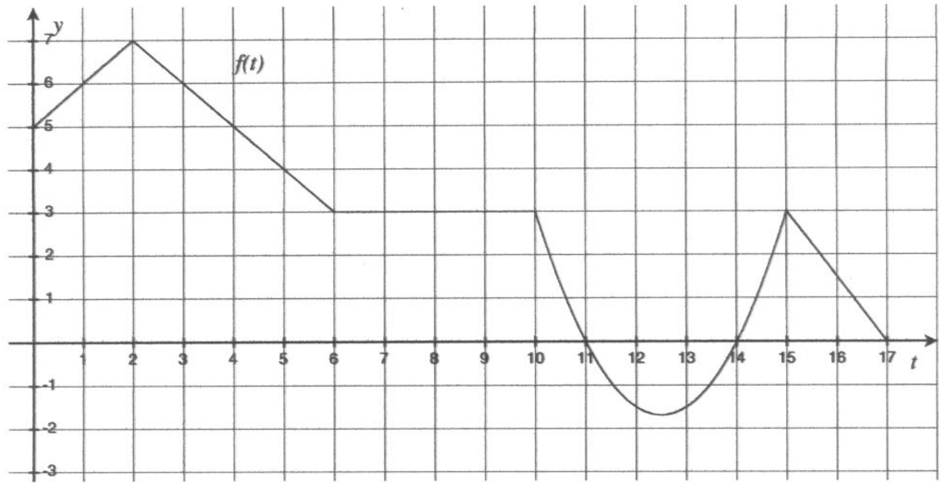
(c) Find all times, t, at which balloon B is ascending 3 ft per minute faster than balloon A.

SOLVE $B'(t) = A'(t) + 3$

ANSWER: $t =$ ________________ minutes

6. (10 points)

The graph below is of the function $y = f(t)$.



Using the graph above, we define a new function

$$A(m) = \int_0^m f(t)\,dt. = \boxed{\text{SIGNED AREA UNDER } f(t) \text{ FROM 0 TO } m}$$

(a) Find the value of t between 0 and 17 at which $\underbrace{f(t)}_{\text{HEIGHT ON GRAPH}}$ has a global maximum.

JUST READ OF LOCATION CORRESPONDING TO HIGHEST POINT

ANSWER: $t =$ ____________

(b) ~~Find all values of~~ m ~~between 0 and 17 at~~ which $\underbrace{A(m)}_{\text{AREA}}$ has a local minimum.

A(m) LOCAL MIN
$\Rightarrow A'(x) = f(x) = 0$
AND $A'(x) = f(x)$ CHANGES FROM POSITIVE TO NEGATIVE

ANSWER: $m =$ ____________

(c) ~~Determine the value of~~ $f'(4)$.

SLOPE ON f(t)
FIND TWO POINTS
COMPUTE SLOPE

ANSWER: $f'(4) =$ ____________

(d) Determine the value of $A'(4)$.

SAME AS $f(4)$ = HEIGHT

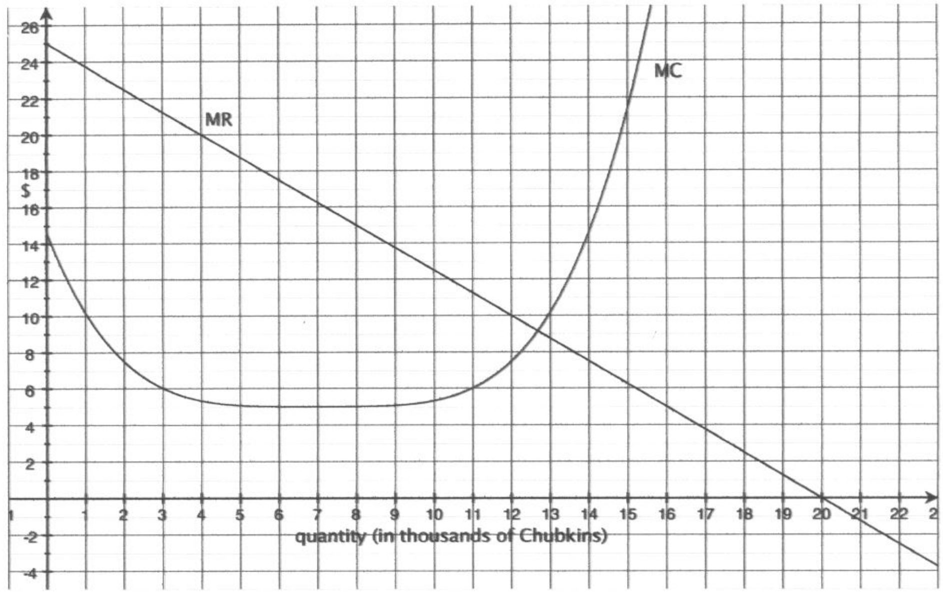
ANSWER: $A'(4) =$ ____________

(e) Compute $\int_6^{10} f(t)\,dt$.

AREA FROM 6 TO 10

ANSWER: $\int_6^{10} f(t)\,dt =$ ____________

7. (10 points) The graphs below are of marginal revenue (MR) and marginal cost (MC) (both in **dollars**) for producing q **thousand** Chubkins.



(a) Determine the increase in total revenue if q increases from 12 to 20 thousand Chubkins.

AREA UNDER MR FROM 12 TO 20

ANSWER: ____________________ thousand dollars

(b) If q changes from 6 to 8 thousand Chubkins, does profit increase or decrease? By how much?

MR > MC SO PROFIT INCREASES!
AREA BETWEEN MR AND MC FROM 6 TO 8
IS HOW MUCH PROFIT GOES UP

ANSWER: Profit (circle one) increases decreases by ______________ thousand dollars.

(c) What is the largest value of total revenue?

OCCURS WHEN $TR'(x) = MR(x) = 0$ SO AT $x = 20$.
FIND AREA UNDER MR FROM 0 TO 20 TO FIND $TR(20)$

ANSWER: ____________________ thousand dollars

(d) If fixed costs are $11.2 thousand dollars, what is the profit earned for selling 2 thousand Chubkins?

$P(2) = TR(2) - TC(2)$
$= TR(2) - VC(2) - FC$
$= \left(\text{AREA BETWEEN MR AND MC FROM 0 TO 2}\right) - FC$

ANSWER: ____________________ thousand dollars

COMPUTE

8. (12 points)

(a) Evaluate: $\int_1^2 x^3 + 4x - 1\, dx$

$$\boxed{\begin{aligned} &= \frac{1}{4}x^4 + 2x^2 - x \Big|_1^2 \\ &= \left(\frac{1}{4}(2)^4 + 2(2)^2 - (2)\right) - \left(\frac{1}{4}(1)^4 + 2(1)^2 - (1)\right) \\ &= \ ? \quad \leftarrow \text{FINISH} \end{aligned}}$$

(b) Evaluate: $\int 3x + \frac{4}{x} - \sqrt{x}\, dx$

$$\boxed{\begin{aligned} &= \int 3x + 4 \cdot \frac{1}{x} - x^{1/2}\, dx \\ &= \frac{3}{2}x^2 + 4\ln(x) - \frac{2}{3}x^{3/2} + C \end{aligned}}$$

FINAL ANSWER

(c) Let $z = \dfrac{t^2 + e^{5t+1}}{t^3 - 4}$. Find $\dfrac{dz}{dt}$. Do not simplify.

QUOTIENT RULE!

DON'T FORGET

$$\boxed{\frac{dz}{dt} = \frac{(t^3-4)(2t + e^{5t+1} \cdot 5) - (t^2 + e^{5t+1})(3t^2)}{(t^3-4)^2}}$$

FINAL ANSWER

(d) Let $f(x, y) = 3x^3y + x^2\ln(x) + y^2$. Find $f_x(x, y)$.

BOTH x's, PRODUCT RULE NEEDED!

$$\boxed{f_x(x,y) = 9x^2y + x^2 \cdot \frac{1}{x} + 2x\ln(x)}$$

FINAL ANSWER

$$\boxed{= 9x^2y + x + 2x\ln(x)}$$

9. (10 points) Let $z = f(x, y) = -3x^2 + 2y^2 + 36x + 12xy$.

   (a) Write out the formulas for $f_x(x, y)$ and $f_y(x, y)$.

   $f_x(x, y) = -6x + 36 + 12y$

   $f_y(x, y) = 4y + 12x$

   (b) Use partial derivatives to tell which of the following numbers is bigger:

   $$\mathbf{A} = \frac{f(1.0001, 1) - f(1,1)}{0.0001} \quad \text{or} \quad \mathbf{B} = \frac{f(1, 1.0001) - f(1,1)}{0.0001}$$

   Justify your answer. (That is, you must show work to get credit.)

   $A \approx f_x(1,1)$ (x IS CHANGING!)

   $B \approx f_y(1,1)$ (y IS CHANGING!)

   COMPUTE! WHICH IS BIGGER

   ANSWER: (circle one) **A** **B**

   (c) Find all points (x, y) which are candidates for local maxima or local minima of $f(x, y)$ or show that there are no such candidates.

   SOLVE SYSTEM (i) $-6x + 36 + 12y \overset{?}{=} 0$

   (ii) $4y + 12x = 0 \Rightarrow 4y = -12x$

   FROM (ii) $y = -3x$

   SUBSTITUTING (ii) INTO (i) GIVES

   $-6x + 36 + 12(-3x) = 0$

   $-6x + 36 - 36x = 0$

   $36 = 42x$

   $x = \frac{36}{42} = \frac{6}{7}$ AND FROM (ii) $y = -3\left(\frac{6}{7}\right) = -\frac{18}{7}$

   ANSWER: (Check one) ☒ The candidate(s) is/are: $(x, y) = \boxed{\left(\frac{6}{7}, \frac{-18}{7}\right)}$

   ☐ There are no such candidates.

   CHECK!