Math 112 - Winter 2018
Exam 1
January 25, 2018

Name: ______________________________

Section: ______________________________

Student ID Number: ______________________________

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- After this cover page, there are 5 problems spanning 4 pages. Please make sure your exam contains all of this material.
- You are allowed to use a Ti-30x IIS Calculator model ONLY (**no other calculators allowed**). And you are allowed one **hand-written** 8.5 by 11 inch page of notes (front and back).
- You must show your work on all problems. The correct answer with no supporting work may result in no credit.
- If you use a guess-and-check, or calculator, method when an algebraic method is available, you may not receive full credit.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- There are **multiple versions** of the exam so if you copy off a neighbor and put down the answers from another version we will know you cheated. Any student found engaging in academic misconduct will receive a score of 0 on this exam. All suspicious behavior will be reported to the academic misconduct board. Sit far away from your study partners and keep your eyes down, don't risk a zero on this exam!
- You have 50 minutes to complete the exam. Budget your time wisely.
SPEND NO MORE THAN 10 MINUTES PER PAGE!

GOOD LUCK!

1. (10 points) Don't do your work in your head, do it on the page (show me all intermediate steps you use). You do NOT have to simplify your final answer. Put a box around your final answer.

   (a) Find $f'(x)$, if $f(x) = (4x^3 - 1)^6 \cdot \left(\frac{x^3}{2} + 7x\right)$.

   (b) Find $\frac{dy}{dx}$, if $y = \frac{4x}{5} + 7\sqrt{x^2 + 4}$.

   (c) The height of a balloon is given by: $h(t) = \frac{t^2 + 4\sqrt{t}}{t^3 + 1}$, where distance is in feet and time is in seconds. Find the instantaneous speed of the balloon at $t = 1$ second. (simplify your numbers and include the units for your final answer).

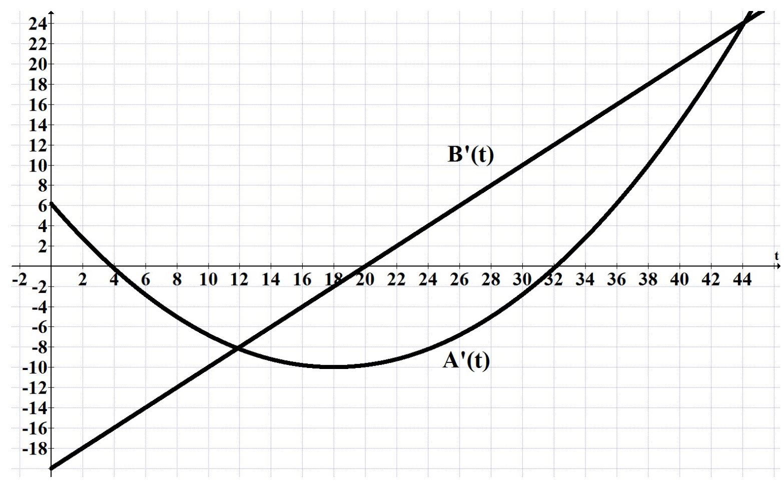
2. (6 pts) Assume $f(x) = \dfrac{45}{x} + 5x$

   (a) Find the second derivative $f''(x)$.

   (b) Solve to find all value(s) of x at which the slope of the tangent line to $f(x)$ is 0.

3. (10 pts)

   Two balloons, A and B, start next to each other at 500 feet and are moving vertically straight up and down. Their **rate of ascent** graphs are shown, where t is in minutes and the rate is in feet/minute. Again, these are the graphs of the *derivatives* of the height functions! Use the graph to answer the following questions as accurately as possible.



   (a) For each part below, circle which quantity is **bigger**.

      i. balloon A height at $t = 0$ or balloon A height at $t = 1$ or They are equal.

      ii. balloon A height at $t = 12$ or balloon B height at $t = 12$ or They are equal.

   (b) Give *all* times when $A(t)$, the height graph for balloon A, has a horizontal tangent.

ANSWER: t = ________________ min

   (c) Find the longest interval over which balloon A is falling and balloon B is rising.

ANSWER: t = ________________ min to t = ________________ min

4. (11 pts) Let $f(x) = 3x^2 - 5x + 1$.

(a) Write out and *expand and completely simplify* the formula for

$$\frac{f(x+h) - f(x)}{h}.$$

ANSWER: $\frac{f(x+h)-f(x)}{h}$ = ____________________

(b) Find the slope of the secant line to $f(x)$ from $x = 2$ to $x = 5$.

ANSWER: secant slope = ____________________

(c) Find the equation for the tangent line to $f(x)$ at $x = 2$.

ANSWER: y = ________________________________

5. (13 pts) You are in charge of marketing a new electronic gadget.
From analyzing the market, you find the demand curve is

$$p = 53 - 2x,$$

where p is price (in dollars per item) and x is in **hundred** items.
From the manufacturer, the total cost function is given by

$$TC(x) = 60 + 5x - 2x^2 + x^3,$$

where x is in hundred items and $TC(x)$ is in hundred dollars. Keep final answers accurate to two digits after the decimal (*i.e.* to the nearest item or nearest dollar).

(a) Find formulas for TR, MR and MC.

$TR(x) =$

$MR(x) =$

$MC(x) =$

(b) Find the largest interval on which total revenue is increasing.

ANSWER: from x= ______________ to x = ______________ hundred items

(c) Find the quantity at which marginal cost is lowest.

ANSWER: x = ______________ hundred items

(d) What selling *price* should you use to maximize profit? (Hint: First, find the quantity that maximizes profit).

ANSWER: selling price = ______________ dollars/item