1. (13 pts) Put a box around your final answer. You do not have to simplify.

(a) Find y' for $y = \left(\ln(t^3+1)\right)^{10}$

$$\boxed{y' = 10\left(\ln(t^3+1)\right)^9 \frac{1}{t^3+1} \cdot 3t^2}$$

(b) Find $f'(x)$ for $f(x) = \frac{1}{2} + 3x + \frac{5}{6e^{\sqrt{x}}}$ $= \frac{1}{2} + 3x + \frac{5}{6} e^{-x^{1/2}}$

$$\boxed{f'(x) = 3 + \frac{5}{6} e^{-x^{1/2}} \cdot \left(-\tfrac{1}{2} x^{-1/2}\right)}$$

(c) Find the general anti-derivative: $\int \frac{\sqrt{x}}{5} - 3e^{2x}\,dx$

$$= \int \frac{1}{5} x^{1/2} - 3e^{2x}\,dx$$

$$= \boxed{\frac{1}{5}\,\frac{2}{3}\,x^{3/2} + \frac{3}{2} e^{2x} + C = \frac{2}{15} x^{3/2} - \frac{3}{2} e^{2x} + C}$$

(d) Evaluate $\int_1^2 x\left(\frac{12}{x^3} + \frac{3}{x}\right) dx$ $= \int_1^2 12x^{-2} + 3\,dx$

$$= \frac{12}{-1} x^{-1} + 3x \Big|_1^2$$

$$= -\frac{12}{x} + 3x \Big|_1^2$$

$$= \left(-\frac{12}{(2)} + 3(2)\right) - \left(-\frac{12}{(1)} + 3(1)\right)$$

$$= (-6+6) - (-12+3)$$

$$= \boxed{9}$$

2. (12 pts) Two balloons are at the same height at $t = 0$. Time, t, is measured in minutes and height is measured in feet. You are given:

$$A'(t) = 15 - \frac{5t}{2} \qquad \text{feet/min} \quad = \text{'RATE of ascent for balloon A'}$$

$$B(t) = \tfrac{1}{3}t^3 - 5t^2 + 24t + 30 \quad \text{feet} \quad = \text{'HEIGHT for balloon B'}$$

(a) Use the fact that $A(0) = B(0)$ to find the formula for $A(t)$ without any undetermined constants.

$A(t) = \int 15 - \frac{5}{2}t\,dt = 15t - \frac{5}{4}t^2 + C$

$A(0) = B(0) = 30$

$\Rightarrow 15(0) - \frac{5}{2}(0)^2 + C = 30 \Rightarrow C = 30$

ANSWER: $\boxed{A(t) = 15t - \frac{5}{4}t^2 + 30}$

(b) Give an interval over which the graph of the height of Balloon B is concave down.

$B'(t) = t^2 - 10t + 24$

$B''(t) = 2t - 10 \overset{?}{=} 0 \Rightarrow t = 5$

$B \quad \frown \quad | \quad \smile$

$B'' \quad - \quad 5 \quad +$

$t =$ 0 to $t =$ 5

(c) Find all times at which Balloon B changes from falling to rising.

$B'(t) = t^2 - 10t + 24 = (t-4)(t-6) = 0 \Rightarrow t = 4$ or $t = 6$

$B \quad \nearrow \quad | \quad \searrow \quad | \quad \nearrow$

$B' \quad + \quad 4 \quad - \quad 6 \quad +$

$t =$ 6 min

(d) Find the lowest and highest altitudes reached by Balloon A from $t = 0$ to $t = 10$.

$A'(t) = 15 - \frac{5}{2}t \overset{?}{=} 0 \Rightarrow 30 - 5t = 0 \Rightarrow t = 6$

$A(0) = 30$

$A(6) = 15(6) - \frac{5}{4}(6)^2 + 30 = 75$

$A(10) = 15(10) - \frac{5}{4}(10)^2 + 30 = 55$

ANSWER: 'lowest altitude' = 30 feet

'highest altitude' = 75 feet

3. (12 pts) You sell items. The functions for marginal revenue and marginal cost (in dollars/item) are given by

$$MR(q) = 7e^{0.02q} \quad \text{and} \quad MC(q) = q^2 - 12q + 124,$$

where q is in thousands of items. You are also told that Fixed Costs are given $FC = 15$ thousand dollars (so $TC(0) = 15$).

(a) Give the functions for Total Revenue and Total Cost (solve for the constants of integration).

$TR(q) = \int 7e^{0.02q}\,dq = \frac{7}{0.02} e^{0.02q} + C = 350e^{0.02q} + C$

$TR(0) = 0 \Rightarrow 350e^0 + C = 0 \Rightarrow C = -350$

$TC(q) = \int q^2 - 12q + 124\,dq = \frac{1}{3}q^3 - 6q^2 + 124q + C$

$TC(0) = 15 \Rightarrow C = 15$

ANSWER: $TR(q) =$ $350e^{0.02q} - 350$

$TC(q) =$ $\frac{1}{3}q^3 - 6q^2 + 124q + 15$

(b) Find the largest and smallest values of Marginal Cost on the interval $q = 0$ to $q = 10$.

$MC'(q) = 2q - 12 \stackrel{?}{=} 0 \Rightarrow q = 6$

$MC(0) = 124$

$MC(6) = (6)^2 - 12(6) + 124 = 36 - 72 + 124 = 88$

$MC(10) = (10)^2 - 12(10) + 124 = 100 - 120 + 124 = 104$

ANSWER: 'smallest value of MC' = 88 dollars/item

'largest values of MC' = 124 dollars/item

(c) Recall: $AC(q) = \frac{TC(q)}{q}$.
Determine if $AC(q)$ is concave up, concave down, or neither at $q = 1$ thousand items. (You must show appropriate derivatives and make correct conclusions to get full credit).

$AC(q) = \frac{1}{3}q^2 - 6q + 124 + 15q^{-1}$

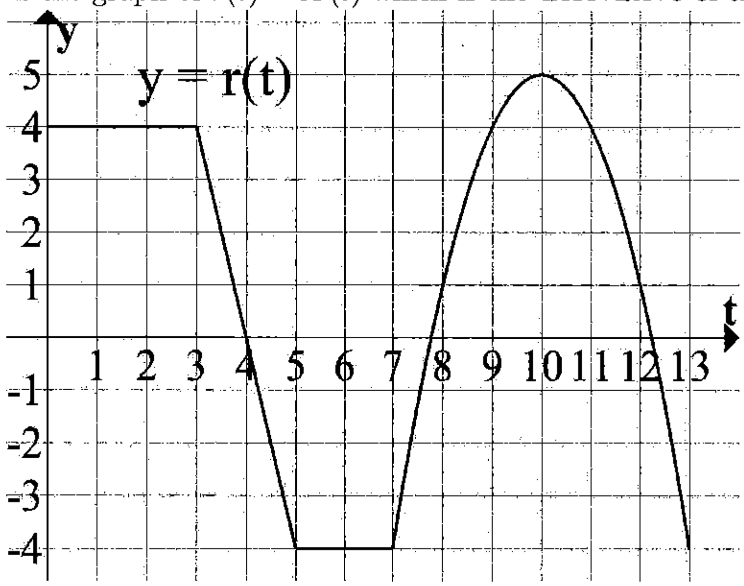
$AC'(q) = \frac{2}{3}q - 6 - 15q^{-2}$

$AC''(q) = \frac{2}{3} + 30q^{-3} = \frac{2}{3} + \frac{30}{q^3}$

$AC''(1) = \frac{2}{3} + 30 > 0$

ANSWER: (Circle one) CONCAVE UP CONCAVE DOWN NEITHER

4. (13 pts) The graph below shows the **rate of ascent**, $r(t)$, at time t for a hot-air balloon. Let $A(t)$ be the function for the height (in feet) of the hot-air balloon at time t minutes. As a reminder, the picture below is the graph of $r(t) = A'(t)$ which is the **derivative** of the altitude function!!



Use the picture to estimate the answers to the questions below as accurately as possible.

(a) Estimate the following:

i. $\int_0^4 r(t)\,dt = 12 + \frac{1}{2}(1)(4) = \boxed{14}$

ii. $\int_3^7 r(t)\,dt = 0 + (2)(-4) = \boxed{-8}$

iii. $A''(4) = r'(4) =$ slope at 4 $= \boxed{-4}$

$(3,4)$, $(4,0)$ $\Big\}$ $\frac{4-0}{3-4} = -4$

(b) Find *all* critical values of $A(t)$ (estimate from the picture).

ANSWER: $t =$ 4, 7, 8, 12.2 min

(c) Give the longest interval of time over which the graph of $A(t)$ is concave up (*remember* the picture above is $A'(t)$). WANT $A''(t)$ positive. $A''(t) = r'(t) =$ slope

POSITIVE SLOPE

ANSWER: $t =$ 7 min to $t =$ 10 min

(d) At time $t = 0$, assume the balloon is 20 feet high. Give the time and the corresponding altitude at which the balloon is highest in the first 7 minutes.

A ↗ ↘

A′ + 4 −

MAX IS AT 4

$\underbrace{\int_0^4 r(t)\,dt}_{14} = A(4) - A(0)$

$14 = A(4) - 20 \Rightarrow A(4) = 34$

ANSWER: $t =$ 4 min
max height $=$ 34 feet