Math 112 - Winter 2018

Final Exam

March 10, 2018

Name: ______________________________

Section: ______________________________

Student ID Number: ______________________________

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|---|---|---|
| 1 | 12 | |
| 2 | 13 | |
| 3 | 13 | |
| 4 | 11 | |
| 5 | 16 | |
| 6 | 12 | |
| 7 | 12 | |
| 8 | 11 | |
| Total | 100 | |

- After this cover page, there are 8 problems spanning 8 pages. Please make sure your exam contains all of this material.
- You are allowed to use a Ti-30x IIS Calculator model ONLY (**no other calculators allowed**). And you are allowed one **hand-written** 8.5 by 11 inch page of notes (front and back).
- You must show your work on all problems. The correct answer with no supporting work may result in no credit.
- If you use a guess-and-check, or calculator, method when an algebraic method is available, you may not receive full credit.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- There are **multiple versions** of the exam so if you copy off a neighbor and put down the answers from another version we will know you cheated. Any student found engaging in academic misconduct will receive a score of 0 on this exam. All suspicious behavior will be reported to the academic misconduct board. Sit far away from your study partners and keep your eyes down, don't risk a zero on this exam!
- You have 2 hours and 50 minutes to complete the exam.

GOOD LUCK!

1. (12 pts)

(a) Find the derivative of $f(x) = x^3 \ln(5x+1) - \dfrac{16}{3x^2} + 5$

(b) Assume the selling price per item is given by $p = \dfrac{32}{3q+2}$ dollars per item.
Find the marginal revenue at $q = 1$ items. (Hint: Write down $TR$ first!)

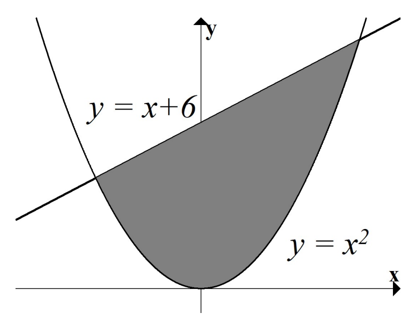
(c) Find the *equation for the tangent line* to $y = \sqrt{e^{5x} + 3}$ at $x = 0$.

2. (13 pts)

(a) Expand and integrate $\int x(3x+2)^2 dx$.

(b) Compute $\int_1^9 \frac{5}{2\sqrt{x}} dx$

(c) Find the area of the region bounded by $y = x + 6$ and $y = x^2$.
(Give your final answer as a decimal)



3. (13 pts) You watch a balloon rise and fall. The height of the balloon (in feet) after $t$ minutes is given by $A(t) = 18t - 3t^2 + 25$.

   (a) Write out and *completely simplify* the formula, in terms of $t$ and $h$, for the change in height from $t$ to $t + h$:

$$A(t+h) - A(t)$$

ANSWER: $A(t+h) - A(t) =$ ______________________

   (b) Find the average rate of ascent for the balloon from $t = 0.5$ min to $t = 2.5$ min. (include units)

ANSWER: ______________ Units = __________

   (c) Find the instantaneous rate of ascent for the balloon at $t = 5$ minutes. (include units)

ANSWER: ______________ Units = __________

   (d) Find the height of the balloon at the moment it changes from rising to falling. (inlude units)

ANSWER: ______________ Units = __________

4. (11 pts) The demand function for a product is $p = \dfrac{77}{x+1}$ and the supply function is $p = 2 + 0.5x$, where $p$ is the price per item, in dollars/item, and $x$ in the number of items.

(a) Find the price and quantity that correspond to market equilibrium.

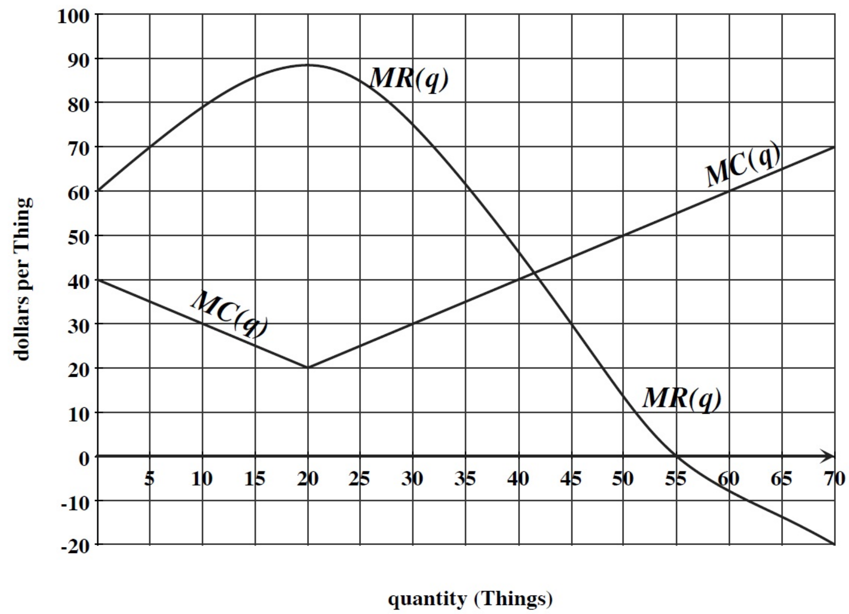
$x =$ __________________ items

$p =$ __________________ dollars/item

(b) Compute the supplier's surplus.

Supplier's surplus = __________________ dollars

5. (16 pts) Below are the graphs of marginal revenue and marginal cost for selling Things:




Also assume that fixed cost is $FC = TC(0) = 25$ dollars.

(a) Estimate as accurately as possible from the graph:

i. $TC(20) =$

ii. $TC'(30) =$

iii. $TC''(10) =$

(b) Give the quantity at which each of the following occur (estimate from the graph):

i. The graph of $MR$ has a local maximum at $q =$

ii. The graph of Profit has a local maximum at $q =$

iii. The graph of $TR$ has a local maximum at $q =$

iv. On the interval $q = 5$ to $q = 33$, the absolute maximum of $TC$ occurs at $q =$

(c) Give the longest interval of time over which the graph of $TR$ is concave up.

$q =$ ________________ to $q =$ ________________

6. (12 pts) Consider the function

$$f(x) = \frac{x^3}{3} - 4x^2 + 12x.$$

(a) Find all critical numbers of $f(x)$ (there are two!).

$x =$ ________________

(b) Classify the critical numbers from the previous part as local max, or local min, or horizontal points of inflection. Clearly indicate your answers and show your reasoning.

(c) Consider the function $D(x) = \frac{f(x)}{x^2}$. Find the absolute minimum and absolute maximum values of $D(x)$ on the interval $x = 1$ to $x = 12$.

Absolute Minimum value = ________________ which occurs at $x =$ __________

Absolute Maximum value = ________________ which occurs at $x =$ __________

7. (12 pts) Let $z = f(x, y) = 14x - 12\ln(y) + \dfrac{2y^4}{x^3}$.

(a) Write out the formulas for $f_x(x, y)$ and $f_y(x, y)$.

$f_x(x, y) =$ ____________________ $f_y(x, y) =$ ____________________

(b) Use a partial derivative to approximate the value of $\dfrac{f(2.0001, 1) - f(2, 1)}{0.0001}$

ANSWER: ________________

(c) If $y = 1$ is fixed, the function $g(x) = f(x, 1)$ is a one variable function of $x$. By showing appropriate calculations, answer the following questions:

i. Is $g(x)$ increasing, decreasing, or neither at $x = 3$?

ANSWER: (circle one) INCREASING DECREASING NEITHER

ii. Is $g(x)$ concave up, concave down, or neither at $x = 3$?

ANSWER: (circle one) CONCAVE UP CONCAVE DOWN NEITHER

8. (11 pts) A company manufactures two products, $A$ and $B$. If $x$ is in thousands of units of $A$ and $y$ is in thousands of units of $B$, then the total cost and total revenue in thousands of dollars are:

$$C(x, y) = 2x^2 - 2xy + y^2 - 8x - 10y + 11$$
$$R(x, y) = 8x + 6y$$

The profit function has one critical point and the maximum profit occurs at this point. Find the maximum profit.

Maximum profit = ____________________ thousand dollars which occurs when

$x =$ ______________ thousand units of $A$ and $y =$ ______________ thousand units of $B$