Math 112 - Winter 2019

Final Exam

March 16, 2019

Name: ______________________________

Section: ______________________________

Student ID Number: ______________________________

1	13	
2	11	
3	14	
4	11	
5	12	
6	14	
7	14	
8	11	
Total	100	

- After this cover page, there are 8 problems spanning 8 pages. Please make sure your exam contains all of this material.
- You are allowed to use a Ti-30x IIS Calculator model ONLY (**no other calculators allowed**). And you are allowed one **hand-written** 8.5 by 11 inch page of notes (front and back).
- You must show your work on all problems. The correct answer with no supporting work may result in no credit.
- If you use a guess-and-check, or calculator, method when an algebraic method is available, you may not receive full credit.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- There are **multiple versions** of the exam so if you copy off a neighbor and put down the answers from another version we will know you cheated. Any student found engaging in academic misconduct will receive a score of 0 on this exam. All suspicious behavior will be reported to the academic misconduct board. Sit far away from your study partners and keep your eyes down, don't risk a zero on this exam!
- You have 2 hours and 50 minutes to complete the exam.

GOOD LUCK!

1. (13 pts) Box your final answer to each of the following.

(a) Let $g(t) = \sqrt{3 + \ln(5t - t^4)}$, find $g'(t)$.

(b) Find $\int \frac{5t}{3} - \frac{7}{8t} + \frac{6}{e^{5t}}\, dt$.

(c) Evaluate $\int_1^{25} \frac{4}{\sqrt{x}}\, dx$.

(d) Let $z = 3x^5 e^{2x} + y\ln(x) + \frac{4}{y^3}$, find BOTH the partial derivatives $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

2. (11 pts) Let $f(x) = 5x - 3x^2 + 1$.

(a) Write out, expand and *completely simplify* the formula, in terms of h, for

$$\frac{f(x+h) - f(x)}{h}.$$

ANSWER: $\frac{f(x+h)-f(x)}{h} =$ ____________________

(b) Find the slope of the secant line to $f(x)$ from $x = 3$ to $x = 3.5$.

ANSWER: ____________________

(c) Find the slope of the tangent line to $f(x)$ at $x = 3$.

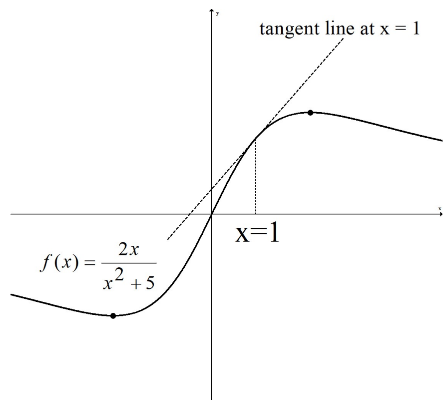
ANSWER: ____________________

3. (14 pts)

(a) Let $g(x) = 2x^2 - 8x + 12$ and $h(x) = \frac{4}{3}x^3 - 400x + 2$.
Find the longest interval on which $g(x)$ is increasing AND $h(x)$ is decreasing.

ANSWER: $x =$ ______________ to $x =$ ______________

(b) Consider $f(x) = \dfrac{2x}{x^2+5}$ (shown below).



i. Find $f'(x)$. (Hint: Quotient rule, check your work!).

ii. Find the following:

A. The height of the graph at $x = 1$ is equal to ______________.

B. The slope of the graph at $x = 1$ is equal to ______________.

C. The equation for the tangent line at $x = 1$ is $y =$ ______________.
(This tangent line is show in the picture).

iii. You can see in the graph that there are two points (marked with black dots) where $f(x)$ has a horizontal tangent. Find the x-coordinates of both these points. (You can leave in exact form or give a decimal approximations).

(List both) $x =$ ______________

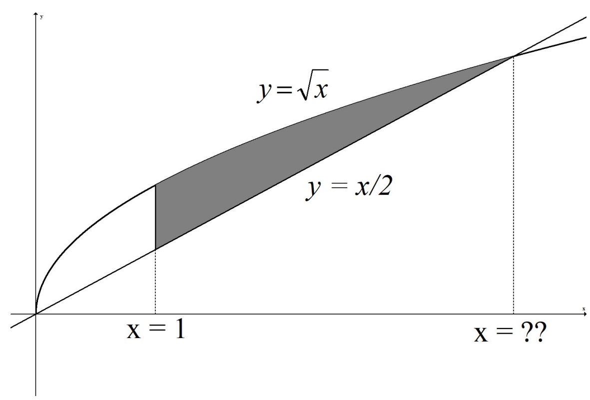
4. (11 pts) The two parts below are not related.

(a) The function $h(x) = 8\ln(x) - 2x + 5$ has one critical number. Find the critical number of $h(x)$ and indiciate if it gives a local maximum, local minimum, or horizontal point of inflection. Show all your work and reasoning (some justification is required).

$x =$ ______________________

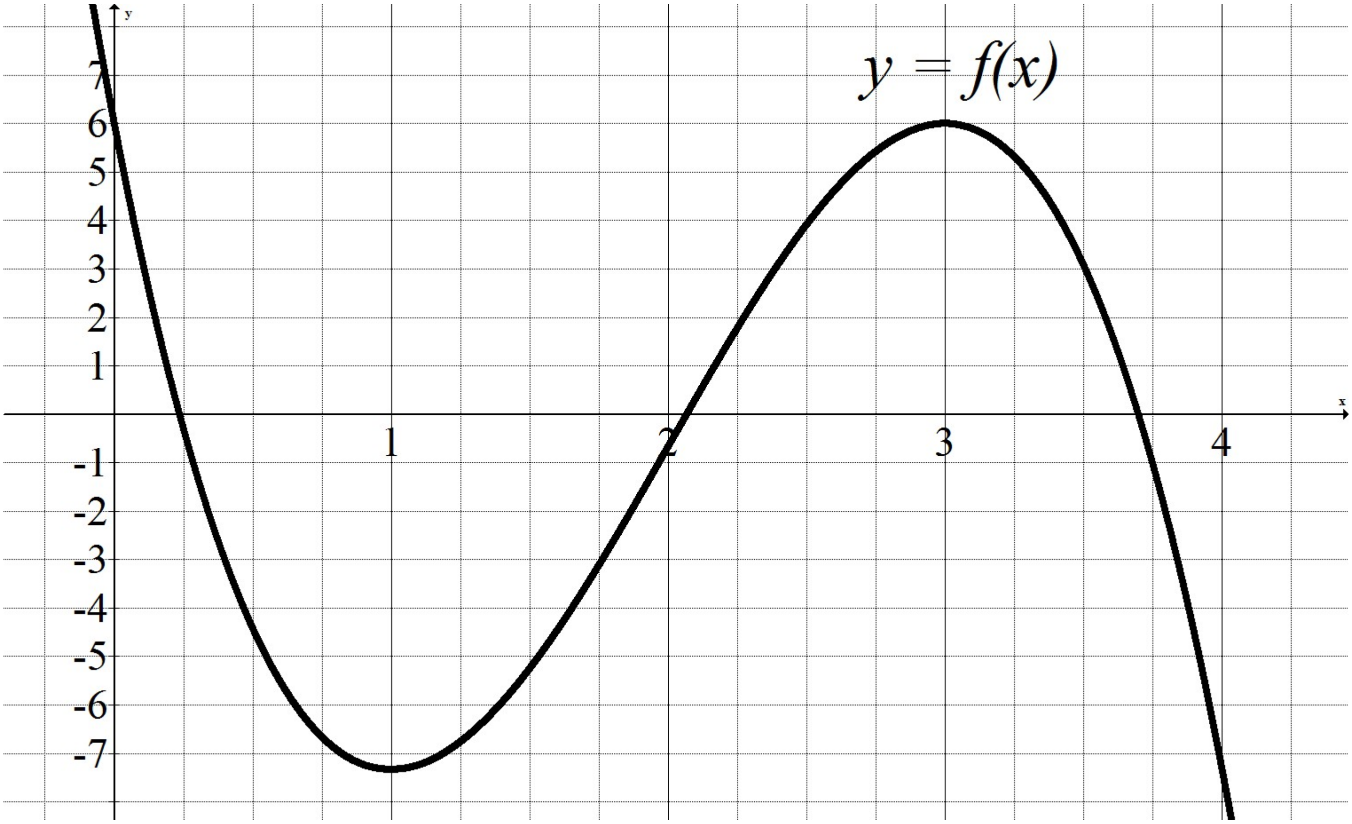
CIRCLE ONE: LOCAL MIN or LOCAL MAX or HORIZ. PT. OF INF.

(b) Find the area of the region bounded by $y = \sqrt{x}$ and $y = \frac{1}{2}x$ and to the right of $x = 1$ (the region is shaded below). Note: You will need to first find their intersection. (You may give your answer as a decimal to three digits after the decimal).



Area = ______________________

5. (12 pts) Consider the graph of $y = f(x)$ below.



As precisely as possible, estimate your answer to the following questions using the graph.

For all parts, assume $A(m) = \int_0^m f(x)dx$.

(a) For each part below, circle the correct answer.

i. The value of $A(1.5)$ is:	POSITIVE	NEGATIVE	ZERO.
ii. The value of $f(1.5)$ is:	POSITIVE	NEGATIVE	ZERO.
iii. The value of $f'(1.5)$ is:	POSITIVE	NEGATIVE	ZERO.
iv. The value of $f''(1.5)$ is:	POSITIVE	NEGATIVE	ZERO.

(b) Find the value(s) of x at which $f'(x) = 0$ and $f''(x)$ is negative.

ANSWER: $x =$ ______________________

(c) Find the value(s) of x at which $A(x)$ has a local maximum.

ANSWER: $x =$ ________________

(d) As accurately as possible, estimate the values of the following from the graph:

i. $A(1) =$

ii. $A'(1) =$

6. (14 pts) For your business you are given the selling price per item and the average cost per item as follows

$$\begin{array}{lll} \text{SELLING PRICE :} & p = 66 - x & \text{dollar/item} \\ \text{AVERAGE COST :} & AC(x) = \dfrac{20}{x} + 81 - 9x + \dfrac{1}{3}x^2 & \text{dollars/item,} \end{array}$$

where x is in ***hundreds*** of items. Keep enough digits to be accurate to the nearest item.

(a) Find the functions for total revenue, total cost, marginal revenue and marginal cost.

$TR(x) =$ ______________________ $MR(x) =$ ______________________

$TC(x) =$ ______________________ $MC(x) =$ ______________________

(b) Find the quantity x at which the second derivative $AC''(x)$ is equal to $\frac{5}{3}$. AND tell me if $AC(x)$ is concave up or concave down at this quantity.

$x =$ ______________ hundred items

CIRCLE ONE: CONCAVE UP or CONCAVE DOWN or NEITHER

(c) Find the *selling price* that corresponds to when profit is maximized (Hint: First find the quantity that maximized profit).

selling price = ______________ dollars/item

7. (14 pts) Let $z = f(x,y) = -x^2 + 6x - 3y^2 + 2y + 2xy + 12$.

(a) Write out the formulas for $f_x(x,y)$ and $f_y(x,y)$.

$f_x(x,y) =$ ______________________ $f_y(x,y) =$ ______________________

(b) Find **all** critical points of $f(x,y)$.

ANSWERS: $(x,y) =$ ______________________

(c) Use a partial derivative to approximate the value of $\dfrac{f(7.0001,2) - f(7,2)}{0.0001}$. (*i.e.* plug an appropriate point in the appropriate partial derivative like you did on the same problem in homework).

ANSWER: ______________________

(d) Find the global minimum and maximum values of the one variable function $z = f(2,y)$ on the interval $y = 0$ to $y = 3$.

ANSWER: Global Min Value: $z =$ ______________________

Global Max Value: $z =$ ______________________

8. (11 pts) A company manufactures two products, A and B. If x is the number of thousands of units of A and y is the number of thousands of units of B, then the total cost and total revenue in thousands of dollars are:

$$C(x, y) = 10x + 5y + x^2 + y^2 + xy$$
$$R(x, y) = 80x + 70y$$

The profit function has one critical point and the maximum profit occurs at this point. Find the maximum profit.

Maximum profit = ________________ thousand dollars which occurs when

$x =$ ____________ thousand units of A and $y =$ ____________ thousand units of B