Math 112 Review 1

This review sheet gives a quick overview of Worksheets 1 - 11 for Math 112. This review sheet may help you remember some of the key points of the course so far. However, this review does not include everything. You are expected to know ALL material from these worksheets for the exam. It is a good idea to look at old exams online at:

http://www.math.washington.edu/ m112/

Also make sure to study the problems at the end of each worksheet.

1. **Worksheet 1, 2, 3 - Slopes of Secants, Slopes of Tangents, and Terminology**

   - You should know what all of the following are:
     total revenue (TR), total cost (TC), profit (P), marginal revenue (MR), marginal cost (MC).
   - "The slope of the secant line from $x = a$ to $x = b$" = $\frac{\text{RISE}}{\text{RUN}}$ = $\frac{f(b)-f(a)}{b-a}$.
   - "The slope of the secant line from $x = m$ to $x = m + h$" = $\frac{f(m+h)-f(m)}{h}$.
     If h is close to zero, then we get an approximation for the slope of the tangent line.
     We experimented with this idea by plugging in numbers like $h = 0.01$ or $h = 0.0001$.
   - *Basic applications*
     - "the (instantaneous) speed of an object" = "the slope of the tangent line to the distance graph"
     - MR = the change in TR from q to q+'one unit'.
       If 'one unit' is 'small' on the graph, then we can say

       $MR \approx$ "the slope of the tangent line to the TR graph"

       $MC \approx$ "the slope of the tangent line to the TC graph".

2. **Worksheet 4, 5, 6 - Derived Graphs**

   - In these sections, we measured the slopes of tangent lines in order to fill in tables. We plotted the slopes of these tangent lines on a different graph and we connected the dots. We called the resulting graph the derived graph.
   - "The Derived Graph of the Distance Functions" = "The Speed Graph"
     "The Derived Graph of Total Revenue" = "The Marginal Revenue Graph"
     "The Derived Graph of Total Cost" = "The Marginal Cost Graph"
   - *We discussed the following general principles*
     - $f'(x)$ crosses the x-axis ($f'(x) = 0$) precisely when $f(x)$ has a horizontal tangent.
     - $f'(x)$ is positive ($f'(x) > 0$) precisely when $f(x)$ is increasing.
     - $f'(x)$ is negative ($f'(x) < 0$)precisely when $f(x)$ is decreasing.
     - You need to be able to answer questions about $f'(x)$, or $f(x)$, by looking at their graphs. This is very important, take the time to really understand these ideas.

3. **Worksheet 7 - The Precise Value for the Slope of the Tangent**

   - We found the precise value of $f'(m)$ as follows:
     (a) Simplify the expression $\frac{f(m+h)-f(m)}{h}$.
     (b) Let h go to zero. $f'(m)$ = what's left over as $h \to 0$.

- Be able to work out these types of problems, even when you are given different information about $\frac{f(m+h)-f(m)}{h}$ (see Problems 3.II, 3.III, 4.II, 5.I, 7.I, 7.II, 7.III)

4. **Worksheet 9 - Derivative Shortcuts**

   - Please, Please, Please Practice Taking Derivatives.
   - *Power Rule*: If $f(x) = x^n$, then $f'(x) = nx^{n-1}$.
   - *Sum Rule*: $(f(x) + g(x))' = f'(x) + g'(x)$.
   - *Coefficient Rule*: $(cf(x))' = cf'(x)$.
   - *Becoming a Derivative Machine*:
     (a) Expand.
     (b) Rewrite Powers.
     (c) Use the Power Rule on each term.
     (d) Simplify your answer.
   - Here is a quick example. Let $y = x\left(\frac{3}{x^2} - 10\sqrt{x} + 4\right) + 77$. Find $\frac{dy}{dx}$.

   $$\begin{aligned} y &= \tfrac{3x}{x^2} - 10x\sqrt{x} + 4x + 77 && \text{Expand} \\ y &= \tfrac{3x^1}{x^2} - 10x^1x^{1/2} + 4x^1 + 77 && \text{Rewrite Powers} \\ y &= 3x^{1-2} - 10x^{1+1/2} + 4x^1 + 77 && \text{Still Rewriting Powers} \\ y &= 3x^{-1} - 10x^{1.5} + 4x^1 + 77 && \text{Finished Rewriting the Powers} \end{aligned}$$

   Now we start to take the derivative:

   $$\begin{aligned} \tfrac{dy}{dx} &= 3(-1x^{-2}) - 10(1.5x^{0.5}) + 4(1x^0) && \text{Use the Power Rule on each term} \\ \tfrac{dy}{dx} &= -3x^{-2} - 15x^{0.5} + 4 && \text{Simplify your answer} \\ \tfrac{dy}{dx} &= -\tfrac{3}{x^2} - 15\sqrt{x} + 4 \end{aligned}$$

   Either of the last two expressions would receive full points on an exam.

5. **Worksheet 10, 11 - Using Derivatives in Business and Finding Max's and Min's**

   - You should be able to use derivatives to find quantity that gives the maximum profit.
     (a) Find formulas for MR and MC, by taking the derivatives of TR and TC.
     (b) Solve MR = MC for q.
     (c) If you are also asked for the maximum profit, then you need to plug q into the profit function.
   - To find the locations where $f(x)$ has horizontal tangents, we compute $f'(x)$. Then we solve $f'(x) = 0$.
   - Once again, you should be able to go back and forth between $f(x)$ and $f'(x)$. You need to know how they are connected and how to use these connections.