CLASSIC OPTIMIZATION PROBLEMS (HOW TO SET UP)

1. *Basic finding numbers examples*:

(A) Homework Question: Find two numbers whose difference is 188 and whose **product is a minimum**.

(B) Old Final Question: The product of two positive numbers is 100. How **small can the sum of one of the numbers plus the square of the other number be**?

2. *Optimizing Area Questions*:

(A) Homework Question: Find the **area of the largest** rectangle that can be inscribed in a right triangle with legs of length 4 cm and 6 cm if two sides of the rectangle lie along the legs.

(B) Old Final Question: A farmer has 136 meters of fencing. She wants to make two rectangular enclosures. One will be a square. The other will have its long side twice as long as its short side (All the possibility that all of the fencing could go to only one of the enclosures.

What dimensions will make the combined total **area as small as possible**?

What dimensions will make the combined total **area as big as possible**?

3. *Minimizing cost/surface area/material used*:

(A) Homework Question: A box with a rectangular base and open top must have a volume of 10 cm^3. The length of one side of the rectangle is twice the width. The material for the base costs $5.00 per square meter and the material for the sides costs $3.00 per square meter. Find dimensions and the corresponding **cost for the cheapest container**.

(B) Old Final Question: An oil refinery is located on the north bank of a straight river that is 2 km wide. A pipeline is to be constructed from the refinery to the storage tanks located on the south bank of the river 6km east of the refinery. The cost of laying pipe is $300,000/km over land and $500,000/km under the river. How should you lay the pipe to **minimize cost**? (A picture was included).

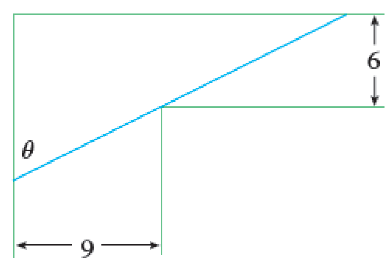
4. *General xy-plane optimization*:

(A) Homework Question: Find the point on the line y = 4x+3 that is **closest to the origin**.

(B) Old Final Question: Find the equation of the line through the point (3,5) that cuts off the **least area from the first quadrant**.

5. *Hints on Two Other Problems from Homework*:

(A) “Find the **longest steel pipe** that will fit around this corner.”
Hint: This is the same as asking for the <u>**minimum length**</u> of this blue line that would touch both walls as shown,
because if it’s longer than this minimum it will get stuck.



(B) “A woman at a point *A* on the shore of a circular lake with radius 2 mi wants to arrive at the point *C* diametrically opposite *A* on the other side of the lake in the shortest possible time (see the figure). She can walk at the rate of 4 mi/h and row a boat at 2 mi/h. For what value of the angle θ shown in the figure will she **minimize her travel time**?“

Hints: Label the center as P. Draw a line segment connecting P and B. Note: this line segment has length 2. Also note, the triangle APB is isosceles, so the angle at B in this triangle is also θ which makes the angle in this triangle 180 - θ – θ = 180 - 2 θ. From this you can conclude that the angle BPC is 2 θ.
You can also assume that the angle ABC is a right angle (because it always will be).
And don’t forget that the arc length around a circle is Arc Length = r θ (in radians).