DERIVATIVE RULES SUMMARY

First, you need to know **all** these well:

$$\frac{d}{dx}\left[x^n\right] = nx^{n-1}$$

$$\frac{d}{dx}\left[e^x\right] = e^x \qquad \frac{d}{dx}\left[a^x\right] = a^x \ln(a)$$

$$\frac{d}{dx}\left[\ln(x)\right] = \frac{1}{x} \qquad \frac{d}{dx}\left[\log_a(x)\right] = \frac{1}{x\ln(a)}$$

$$\frac{d}{dx}\left[\sin(x)\right] = \cos(x) \qquad \frac{d}{dx}\left[\cos(x)\right] = -\sin(x)$$

$$\frac{d}{dx}\left[\tan(x)\right] = \sec^2(x) \qquad \frac{d}{dx}\left[\cot(x)\right] = -\csc^2(x)$$

$$\frac{d}{dx}\left[\sec(x)\right] = \sec(x)\tan(x) \qquad \frac{d}{dx}\left[\csc(x)\right] = -\csc(x)\cot(x)$$

$$\frac{d}{dx}\left[\sin^{-1}(x)\right] = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\left[\cos^{-1}(x)\right] = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left[\tan^{-1}(x)\right] = \frac{1}{1+x^2} \qquad \frac{d}{dx}\left[\cot^{-1}(x)\right] = -\frac{1}{1+x^2}$$

$$\frac{d}{dx}\left[\sec^{-1}(x)\right] = \frac{1}{x\sqrt{x^2-1}} \qquad \frac{d}{dx}\left[\csc^{-1}(x)\right] = -\frac{1}{x\sqrt{x^2-1}}$$

Second, you need to be able to recognize the given form of the function/equation in question:

Product: Does it look like $y = f(x)g(x)$? $\quad \frac{d}{dx}\left[f(x)g(x)\right] = f(x)g'(x) + f'(x)g(x)$.

Quotient: Does it look like $y = \frac{f(x)}{g(x)}$? $\quad \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$.

Chain: Does it look like $y = f(g(x))$? $\quad \frac{d}{dx}\left[f(g(x))\right] = f'(g(x))g'(x)$.

Logarithmic: Does it look like $y = f(x)^{g(x)}$? Rewrite as $\ln(y) = g(x)\ln(f(x))$, use implicit.

Implicit: Can't solve for y in terms of x? $\quad \frac{d}{dx}\left(F(x,y) = 0\right)$ (Think of y as a function of x)

Parametric: Given $x = x(t)$, $y = y(t)$? $\quad \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$.

PRECALCULUS AND SOLVING REVIEW

Various solving and precalculus questions have been coming up in office hours. Remember that I have precalculus review sheets posted online from the first week of the quarter. Also remember that chapter 1 of your book has some detailed review materials (as does the appendix). But here are a few quick things based on recent questions.

Polynomials: We expect you to be able to solve linear and quadratic equations: $ax + b = 0$ and $ax^2 + bx + c = 0$. If a polynomial equation has a cubic or higher power and you are being asked to solve, then there must be some way to factor or simplify. If you already know one solution, note that you can divide out that solution. For example, if we tell you that $t = 1$ is a solution to $t^3 - 2t + 1 = 0$, then you can divide by $t^3 - 2t + 1$ by $t - 1$ (using normal long division) and conclude $t^3 - 2t + 1 = (t-1)(t^2 + t - 1) = 0$. If you have never done this, then ask me in office hours.

Powers/Roots: To solve $x^k = b$ for x, you take a 'root'. If k is an even integer, then there are two solutions $x = \pm b^{1/k}$. If k is an odd integer, then there is one solution $x = b^{1/k}$.

Exponentials/Logarithms: You should know what the graph of $y = e^x$ and $y = \ln(x)$ look like. And you should know they are inverses. You should be very familiar with all the facts:

1. $\ln(e^x) = x$ and $e^{\ln(x)} = x$.
2. $\ln(ab) = \ln(a) + \ln(b)$, $\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$, and, most importantly, $\ln(a^b) = b\ln(a)$.
3. Note a couple values:
   - $e^0 = 1$ which is the same as saying $\ln(1) = 0$.
   - $e^1 = e$ which is the same as saying $\ln(e) = 1$.

Trig/Inverse Trig: You should know the definitions of all 6 trig and all 6 inverse trig functions. Know where they are defined and what they give.

$\sin(x) = \frac{\text{opp}}{\text{hyp}}$	$\cos(x) = \frac{\text{adj}}{\text{hyp}}$
$\tan(x) = \frac{\sin(x)}{\cos(x)}$	$\cot(x) = \frac{\cos(x)}{\sin(x)}$
$\sec(x) = \frac{1}{\cos(x)}$	$\csc(x) = \frac{1}{\sin(x)}$

θ	$\cos(\theta)$	$\sin(\theta)$
0	1	0
$\pi/6$	$\sqrt{3}/2$	$1/2$
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$
$\pi/3$	$1/2$	$\sqrt{3}/2$
$\pi/2$	0	1

You should also know the equivalent values in the other quadrants of the unit circle. For example, you should know that $\sin(2\pi/3) = \sin(\pi/3) = \sqrt{3}2$ and $\cos(2\pi/3) = -\cos(\pi/3) = -1/2$.
Here are a few quick examples where we use inverse trig functions to solve:

- To solve $\sin(x) = 0.4$, we get the solution that is between $-\pi/2$ and $\pi/2$ by using $x = \sin^{-1}(0.4)$. To get other solutions, we use the graph of $y = \sin(x)$. (Note: The other solutions are $x = 2k\pi + \sin^{-1}(0.4)$ and $x = 2k\pi + (\pi - \sin^{-1}(0.4))$ for any integer k).
- To solve $\cos(x) = -0.2$, we get the solutions that is between 0 and π by using $x = \cos^{-1}(-0.2)$. To get other solutions, we use the graph of $y = \cos(x)$. (Note: The other solutions are $x = 2k\pi + \cos^{-1}(-0.2)$ and $x = 2k\pi - \cos^{-1}(-0.2)$).
- To solve $\tan(x) = 10$, we get the solution that is strictly between $-\pi/2$ and $\pi/2$ by using $x = \tan^{-1}(10)$. To get other solutions, we use the graph of $y = \tan(x)$. (Note: The other solutions are $x = k\pi + \tan^{-1}(10)$ for any integer k).