Limits Practice

With the techniques we have developed, we can now evaluate many different types of limits. Below is a large collection of limit problems each pulled directly from the old exam archives. For each problem, evaluate the limit and give justification. See if you can do these quickly. Solutions are posted online.

1. $\lim_{x \to A} \frac{\frac{1}{x} - \frac{1}{A}}{x - A} \cdot \frac{Ax}{Ax}$ $\frac{0}{0}$ ✓

$= \lim_{x \to A} \frac{A - x}{(x-A)Ax}$

$= \lim_{x \to A} \frac{-\cancel{(x-A)}}{\cancel{(x-A)}Ax}$

$= \lim_{x \to A} \frac{-1}{Ax}$

$= \frac{-1}{A \cdot A} = \boxed{-\frac{1}{A^2}}$

2. $\lim_{x \to 8} \frac{\sqrt{x-4}+2}{x-3}$

$= \frac{\sqrt{8-4}+2}{8-3}$

$= \boxed{\frac{4}{5}}$

continuous at $x=8$

3. $\lim_{x \to -2^-} \frac{|x+2|}{x^2+7x+10}$ $\frac{0}{0}$ ✓

For $x < -2$, $|x+2| = -(x+2)$

$= \lim_{x \to -2^-} \frac{-\cancel{(x+2)}}{\cancel{(x+2)}(x+5)}$

$= \lim_{x \to -2^-} \frac{-1}{x+5}$

$= \frac{-1}{-2+5} = \boxed{-\frac{1}{3}}$

4. $\lim_{x \to \infty} \left(x - \sqrt{x^2+4x}\right)$ conj.

$= \lim_{x \to \infty} \frac{x^2 - (x^2+4x)}{x + \sqrt{x^2+4x}}$ $\frac{\infty}{\infty}$ ✓

$= \lim_{x \to \infty} \frac{-4x}{x + \sqrt{x^2+4x}} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$

$= \lim_{x \to \infty} \frac{-4}{1 + \sqrt{1 + 4/x}}$ NOTE: $x = \sqrt{x^2}$ for $x > 0$

$= \frac{-4}{1+\sqrt{1+0}} = -\frac{4}{2} = \boxed{-2}$

5. $\lim_{x \to 2} \frac{x^2+x-6}{x-2}$ $\frac{0}{0}$ ✓

$= \lim_{x \to 2} \frac{\cancel{(x-2)}(x+3)}{\cancel{(x-2)}}$

$= 2 + 3 = \boxed{5}$

6. $\lim_{x \to 3} \frac{x^2-4x+3}{x^2+4x-21}$ $\frac{0}{0}$ ✓

$= \lim_{x \to 3} \frac{(x-1)\cancel{(x-3)}}{(x+7)\cancel{(x-3)}}$

$= \frac{3-1}{3+7} = \frac{2}{10} = \boxed{\frac{1}{5}}$

7. $\lim_{x \to 2} \frac{\sqrt{x^2+4} - \sqrt{8}}{x^2-4}$ $\frac{0}{0}$ ✓ conj.

$= \lim_{x \to 2} \frac{x^2+4-8}{(x^2-4)(\sqrt{x^2+4}+\sqrt{8})}$

$= \lim_{x \to 2} \frac{\cancel{(x^2-4)}}{\cancel{(x^2-4)}(\sqrt{x^2+4}+\sqrt{8})}$

$= \frac{1}{\sqrt{2^2+4}+\sqrt{8}}$

$= \boxed{\frac{1}{2\sqrt{8}} = \frac{1}{4\sqrt{2}} = \frac{\sqrt{2}}{8}}$

ALL SAME ANSWER

8. $\lim_{\theta \to 0} \frac{\sin(13\theta)}{4\theta} \cdot \frac{13}{13}$ $\frac{0}{0}$ ✓

$= \lim_{\theta \to 0} \frac{13}{4} \underbrace{\frac{\sin(13\theta)}{13\theta}}_{\text{"goes to 1"}}$

$= \boxed{\frac{13}{4}}$

9. $\lim_{t \to \pi/2} \frac{\sin(t) + \sqrt{\sin^2(t) + 2\cos^2(t)}}{2\cos^2(t)}$ $\frac{2}{0}$

numerator $\to 1 + \sqrt{1+0} = +2$ (+)

denominator $\to 2(0)^2 = 0$

NOTE: $2(\cos(t))^2$ WILL ALWAYS BE ≥ 0. (+)

$\boxed{+\infty}$

10. $\lim_{x\to 0} \frac{x^3-8}{\cos^2(x)}$

$= \frac{-8}{1}$

$= \boxed{-8}$

continuous at $x=0$

11. $\lim_{t\to 0} \left(\frac{1}{t\sqrt{1+t}} - \frac{1}{t} \right)$ $\frac{0}{0}$ ✓

$= \lim_{t\to 0} \frac{1-\sqrt{1+t}}{t\sqrt{1+t}}$

conj.

$= \lim_{t\to 0} \frac{1-(1+t)}{t\sqrt{1+t}\,(1+\sqrt{1+t})}$

$= \lim_{t\to 0} \frac{-\cancel{t}}{\cancel{t}\sqrt{1+t}\,(1+\sqrt{1+t})}$

$= \frac{-1}{\sqrt{1}\,(1+\sqrt{1})} = \boxed{-\frac{1}{2}}$

12. $\lim_{r\to 0} \frac{3r^2}{3-\sqrt{9-r^2}}$ $\frac{0}{0}$ ✓

$= \lim_{r\to 0} \frac{3r^2(3+\sqrt{9-r^2})}{9-(9-r^2)}$

conj.

$= \lim_{r\to 0} \frac{3\cancel{r^2}(3+\sqrt{9-r^2})}{\cancel{r^2}}$

$= 3(3+\sqrt{9}) = \boxed{18}$

13. $\lim_{x\to 0} \frac{e^x\sin(3x)+5\sin(3x)}{x}$ $\frac{0}{0}$ ✓

$= \lim_{x\to 0} (e^x+5)\,\frac{\sin(3x)}{x}\,\frac{3}{3}$

$= \lim_{x\to 0} 3(e^x+5)\,\underbrace{\frac{\sin(3x)}{3x}}_{\text{goes to } 1}$

$= 3(\underbrace{e^0}_{=1}+5)\cdot 1$

$= 3(6) = \boxed{18}$

14. $\lim_{x\to 2^-} \frac{e^x}{2-x}$ $\frac{e^2}{0}$

numerator: $e^2 > 0$ $+$

denominator: $2-x > 0$ $+$
for $x<2$

$\lim_{x\to 2^-} \frac{e^x}{2-x} = \boxed{+\infty}$

15. $\lim_{x\to\infty} \left(x-\sqrt{x^2+3}\right)$ $\frac{\infty}{\infty}$ ✓

$= \lim_{x\to\infty} \frac{x^2-(x^2+3)}{x+\sqrt{x^2+3}}$

conj.

$= \lim_{x\to\infty} \frac{-3}{x+\sqrt{x^2+3}}\;\frac{\frac{1}{x}}{\frac{1}{x}}$

$= \lim_{x\to\infty} \frac{-3/x}{1+\sqrt{1+3/x^2}}$

NOTE: $\frac{1}{x} = \frac{1}{\sqrt{x^2}}$ for $x>0$

$= \frac{0}{1+\sqrt{1+0}} = \frac{0}{2} = \boxed{0}$

16. $\lim_{t\to 3} \frac{\frac{1}{3}-\frac{3}{t^2}}{t-3}\;\frac{3t^2}{3t^2}$ $\frac{0}{0}$ ✓

$= \lim_{t\to 3} \frac{t^2-9}{(t-3)\,3t^2}$

$= \lim_{t\to 3} \frac{\cancel{(t-3)}(t+3)}{\cancel{(t-3)}\,3t^2}$

$= \frac{(3+3)}{3(3)^2}$

$= \frac{6}{27} = \boxed{\frac{2}{9}}$

17. $\lim_{x\to 2} \frac{\sqrt{x^2+12}}{(x-2)^2}$ $\frac{4}{0}$

numerator: $\sqrt{(2)^2+12} > 0$ $+$

denominator: $(x-2)^2 > 0$ $+$
for all x around 2

$\lim_{x\to 2} \frac{\sqrt{x^2+12}}{(x-2)^2} = \boxed{+\infty}$

18. $\lim_{x\to 0^+} \frac{1+x}{e^{x^2-x}-1}$ $\frac{1}{0}$

numerator: $1+0=1$ $+$

denominator:

For x slightly larger than 0, you get $x^2-x = x(x-1)$ is negative, ($+\cdot-$)

AND SO $e^{(x^2-x)}$ will be smaller than 1.

Thus, $e^{x^2-x}-1$ WILL BE NEGATIVE $-$

$\lim_{x\to 0^+} \frac{1+x}{e^{(x^2-x)}-1} = \boxed{-\infty}$

22. $\lim_{x\to 3} \frac{\sqrt{x^2-9}}{\sqrt{2x-6}}$ $\frac{0}{0}$ ✓

$= \lim_{x\to 3} \sqrt{\frac{x^2-9}{2x-6}}$

$= \lim_{x\to 3} \sqrt{\frac{\cancel{(x-3)}(x+3)}{2\cancel{(x-3)}}}$

$= \sqrt{\frac{3+3}{2}} = \boxed{\sqrt{3}}$

23. $\lim_{x\to 0^-} \frac{|x-|x||}{|2x-|x||}$ $\frac{0}{0}$ ✓

For $x<0$, $|x| = -x$

$= \lim_{x\to 0^-} \frac{|x-(-x)|}{|2x-(-x)|}$

$= \lim_{x\to 0^-} \frac{|2x|}{|3x|}$

$= \lim_{x\to 0^-} \frac{-2x}{-3x}$

$= \frac{-2}{-3} = \boxed{2/3}$

24. $\lim_{x\to 0} \frac{\sin(2x)}{x}$ $\frac{2}{2}$ $\frac{0}{0}$ ✓

$= \lim_{x\to 0} 2 \underbrace{\frac{\sin(2x)}{2x}}_{\text{goes to } 1}$

$= 2\cdot 1 = \boxed{2}$

25. $\lim_{x\to 3^+} \frac{x+e^x}{(3-x)e^x}$ $\frac{3+e^3}{0}$

numerator: $3+e^3$ IS POSITIVE $+$

denominator: For $x>3$,

$3-x$ IS NEGATIVE

SO $(3-x)e^x$ IS NEGATIVE $-$

$\lim_{x\to 3^+} \frac{x+e^x}{(3-x)e^x} = \boxed{-\infty}$

26. $\lim_{t\to 0} \left(\frac{1}{2t\sqrt{1+2t}} - \frac{1}{2t}\right)$

$= \lim_{t\to 0} \frac{1-\sqrt{1+2t}}{2t\sqrt{1+2t}}$ $\frac{0}{0}$ ✓

$= \lim_{t\to 0} \frac{1-(1+2t)}{2t\sqrt{1+2t}\,(1+\sqrt{1+2t})}$ } conj.

$= \lim_{t\to 0} \frac{-2t}{2t\sqrt{1+2t}\,(1+\sqrt{1+2t})}$

$= \frac{-1}{\sqrt{1}\,(1+\sqrt{1})} = \boxed{-\frac{1}{2}}$

27. $\lim_{x\to\infty} \frac{3x^{10}+x^8+3}{x^{10}-3x^6+2}$ $\frac{\frac{1}{x^{10}}}{\frac{1}{x^{10}}}$ $\frac{\infty}{\infty}$ ✓

$= \lim_{x\to\infty} \frac{3+\frac{1}{x^2}+\frac{3}{x^{10}}}{1-\frac{3}{x^4}+\frac{2}{x^{10}}}$

$= \frac{3+0+0}{1-0+0}$

$= \boxed{3}$

28. $\lim_{x\to 1} \frac{e^x+7}{(x-1)^3}$ $\frac{e+7}{0}$

numerator: $e+7$ IS POSITIVE $+$

denominator:

For $x<1$, $(x-1)^3$ IS NEGATIVE $-$

For $x>1$, $(x-1)^3$ IS POSITIVE $+$

THUS,

$\lim_{x\to 1^-} \frac{e^x+7}{(x-1)^3} = -\infty$

$\lim_{x\to 1^+} \frac{e^x+7}{(x-1)^3} = \infty$

$\boxed{\lim_{x\to 1} \frac{e^x+7}{(x-1)^3} \text{ DNE}}$

29. $\lim_{x\to 5} \frac{(x^2-25)\sin(x)}{(x-5)\cos(x)}$ $\frac{0}{0}$ ✓

$= \lim_{x\to 5} \frac{\cancel{(x-5)}(x+5)\sin(x)}{\cancel{(x-5)}\cos(x)}$

$= (5+5)\frac{\sin(5)}{\cos(5)}$

$= \boxed{10\frac{\sin(5)}{\cos(5)} = 10\tan(5)}$

same answer

30. $\lim_{x\to 0} \frac{x}{\sqrt{x+4}-2}$ $\frac{0}{0}$ ✓

$= \lim_{x\to 0} \frac{x(\sqrt{x+4}+2)}{x+4-4}$ conj.

$= \lim_{x\to 0} \frac{\cancel{x}(\sqrt{x+4}+2)}{\cancel{x}}$

$= \sqrt{4}+2$

$= \boxed{4}$