Week 1 Overview

My reviews and review sheets are not meant to be your only form of studying. It is vital to your success on the exams that you carefully go through and understand ALL the homework problems, worksheets and lecture material. Hopefully this review sheet will remind you of some of the key ideas of these sections.

10.1: Parametric Equations

1. *Parametric equations* for motion in the plane are any set of equations $x = f(t)$ and $y = g(t)$ that give the x and y coordinates separately in terms of a parameter t.

2. To graph a parametric equation:

   (a) Select various values of t and evaluate the functions to find the corresponding points (x, y).

   (b) Plot these points and indicate the corresponding time and direction of movement.

3. To find the equation for the *curve*, or path, traced out by the motion, try to eliminate the parameter by either.

   (a) Solving for t in one equation and substituting in the other.

   (b) Use trig identities to combine the equations (namely $\sin^2(\theta) + \cos^2(\theta) = 1$).

4. Two Particular Scenarios:

   - Uniform Linear Motion: Motion at a constant speed at a straight line is modeled by the parametric equations $x = a + bt$ and $y = c + dt$,
     where $b = \frac{\Delta x}{\Delta t}$ = 'the velocity in the x-direction' and $d = \frac{\Delta y}{\Delta t}$ = 'the velocity in the y-direction' and (a, c) = 'the locations at time $t = 0$'.
   - Circular Motion: An object moving on a circular path at a constant angular speed is modeled by the parametric equations

     $$x = x_c + r\cos(\theta_0 + \omega t) \qquad y = y_c + r\sin(\theta_0 + \omega t),$$

     where (x_c, y_c) = 'the center of the circle', r = 'the radius',
     θ_0 = the initial angle measured from the positive x-axis
     $\omega = \frac{\Delta \text{Angle}}{\Delta t}$ = 'the angular speed typically in units of rad/sec' (counterclockwise motion is positive and clockwise motion is negative).

2.1: Tangent Lines and Velocity

1. Understand conceptually how a tangent line at a point on a curve is the line that is approached as secant lines are drawn between the given point and 'nearby' points on the curve (this will be made more precise later).

2. Understand conceptually that instantaneous velocity of an object is the same as the slope of the tangent line to the total distance vs. time graph for that object.

2.2: The Limit of a Function

1. We write $\lim_{x\to a} f(x) = L$ to mean 'the limit of $f(x)$, as x approaches a, equals L'. Informally, this means that as x gets closer to a, $f(x)$ gets closer to L. More precisely, for any arbitrarily small interval around L, it is possible to find a small enough interval around a such that if x is within the small interval about a, that the corresponding output $f(x)$ is within the small interval about L.

2. The limit $\lim_{x\to a} f(x) = L$ says nothing about what is happening at the exact value a, it says instead that as x values get closer from the left and right sides of a, that the corresponding $y = f(x)$ values get close to L.

3. We write $\lim_{x\to a^-} f(x) = L$ to mean 'the limit of $f(x)$, as x approaches a from the left, equals L'. This means that as x gets closer to a from smaller values than a, that $f(x)$ gets closer to L.

4. We write $\lim_{x\to a^+} f(x) = L$ to mean 'the limit of $f(x)$, as x approaches a from the right, equals L'. This means that as x gets closer to a from larger values than a, that $f(x)$ gets closer to L.

5. We can only write $\lim_{x\to a} f(x) = L$ if $\lim_{x\to a^-} f(x) = L$ and $\lim_{x\to a^+} f(x) = L$.

6. We write $\lim_{x\to a} f(x) = \infty$ to mean that $f(x)$ gets arbitrarily large in the positive direction as x approaches a (from both sides). This can also be defined for one-sided limits.

7. We write $\lim_{x\to a} f(x) = -\infty$ to mean that $f(x)$ gets arbitrarily large in the negative direction as x approaches a (from both sides). This can also be defined for one-sided limits.

8. We will learn more limit methods as we work through chapter 2. Here our current methods for evaluating a limit:

   - Make a table, by evaluate the function at several values 'near' the value $x = a$. See if you can see the value that the function is approaching.
   - Make an accurate picture of the function and see if you can deduce the limit at x approaches a.