Week 2 Overview

My reviews and review sheets are not meant to be your only form of studying. It is vital to your success on the exams that you carefully go through and understand ALL the homework problems, worksheets and lecture material. Hopefully this review sheet will remind you of some of the key ideas of these sections.

2.3: Limit Laws

1. Know the various limits laws on pages $99 - 101$ and how to use them in practice.

2. Understand the various algebraic methods to manipulate indeterminate forms of limits. Here are some common strategies (see problems $2.3/11 - 30$ for more practice):

   - If you know the function is continuous at the value, plug it in.
   - Factor and simplify.
   - Expand and simplify.
   - Combine/Simplify fractions.
   - Use conjugates for radicals.

3. Understand the Squeeze Theorem and how to use it. Namely that if $f(x) \leq g(x) \leq h(x)$ for x near a (except possibly at a) and $\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$, then $\lim_{x \to a} g(x) = L$. This can be quite useful as we have seen in various examples (especially involving $\sin(x)$ and $\cos(x)$). See problems $2.3/33 - 38$ for a few basic examples.

4. Be able to think about limits from the left and right. Often absolute values give good practice with this, so try problems $2.3/39 - 44$ for practice.

2.5: Continuity

1. A function, $f(x)$, is **continuous** at a if $\lim_{x \to a} f(x) = f(a)$. This requires the following three facts about f at a:

   (a) $f(a)$ is defined (that is, a is in the domain of f).
   (b) $\lim_{x \to a} f(x)$ exists.
   (c) $\lim_{x \to a} f(x) = f(a)$.

2. If $\lim_{x \to a^-} f(x) = f(a)$, then we say $f(x)$ is **continuous from the left** at a.

3. If $\lim_{x \to a^+} f(x) = f(a)$, then we say $f(x)$ is **continuous from the right** at a.

4. Visually, we informally say a function is continuous if you can draw it without ever lifting your pencil from the paper.

5. Know the domains of all the standard function (polynomials, rational functions, exponentials, trig, etc). These functions are always continuous *on their domains*, so your first job is often to recognize the values outside the domain.

6. If a function is continuous at a, then you can find the limit as $x \to a$ by simply evaluating the function at a. So our first attempt in evaluating a limit is often to evaluate and check continuity.

7. The *Intermediate Value Theorem* states a very basic, but important, consequence of continuity. If $f(x)$ is continuous on $[a, b]$ and $f(a) \leq K \leq f(b)$, then there exists a number c with $a \leq c \leq b$ such that $f(c) = K$. (As a quick example, if $f(1) = 7$ and $f(3) = 50$ AND $f(x)$ is continuous on $[1, 3]$, then all y-values between 7 and 50 must be 'hit' by the function $f(x)$ at some x values between 1 and 3).

2.6: Limits at Infinity

1. We define $\lim_{x \to \infty} f(x) = L$ to mean that $f(x)$ can be made arbitrarily close to L by taking x sufficiently large (positive).

2. We define $\lim_{x \to -\infty} f(x) = L$ to mean that $f(x)$ can be made arbitrarily close to L by taking x sufficiently large (negative).

3. If $\lim_{x \to \infty} f(x) = L$ or $\lim_{x \to -\infty} f(x) = L$, then we say $f(x)$ has a horizontal tangent at $y = L$.

4. The evaluation of infinite limits is made significantly easier if we have a basic list of limits to work from. Here are a few that are nice to know:

   - If $r > 0$, then $\lim_{x \to \pm\infty} x^{-r} = \lim_{x \to \pm\infty} \dfrac{1}{x^r} = 0$. (in the case of $-\infty$ we require that r is such that x^r is defined).

   - $\lim_{x \to \infty} \dfrac{1}{e^x} = \lim_{x \to \infty} e^{-x} = 0$.

   - $\lim_{x \to \infty} \tan^{-1}(x) = \pi/2$ and $\lim_{x \to -\infty} \tan^{-1}(x) = -\pi/2$.

5. To find an infinite limit (for extra practice see $2.6/15 - 36$).

   (a) Try multiplying by $\frac{1}{x^r}$ where r is the highest power in the rational function (if there is a square root, halve the largest power under the radical). Simplify and use the known result above.

   (b) Try multiplying by $\frac{1}{e^{rx}}$ and see if that helps.

   (c) Use conjugates with sums/differences involving roots.

   (d) Use the Squeeze Theorem (See problems $2.6/53, 57$).

6. Warning: $\infty - \infty \neq 0$, $\infty \cdot 0 \neq 0$, etc. It does not make sense to manipulate the symbol ∞ in this way (and it does not give the right answer). In such situation you must consider the entire behavior of the expressions and use algebra to manipulate the expression into one of the forms above.