

Week 7 Overview

My reviews and review sheets are not meant to be your only form of studying. It is vital to your success on the exams that you carefully go through and understand ALL the homework problems, worksheets and lecture material. Hopefully this review sheet will remind you of some of the key ideas of these sections.

4.1: Critical Points and Absolute Max/Min

1. We defined a **critical number** of $f(x)$ on domain D to be any number $x = c$ on the domain D such that either
 - (a) $f'(c) = 0$, or
 - (b) $f'(c)$ does not exist.
2. We discussed that any continuous function on a closed interval must have an absolute (global) maximum and an absolute (global) minimum on that interval (The Extreme Value Theorem). And we discovered that all absolute max/min occur either at a critical number or at an endpoint. This gave us the absolute max/min method:

To find the absolute max/min of a continuous function $f(x)$ on a closed interval:

- (a) Find the critical numbers.
- (b) Evaluate $f(x)$ at the critical numbers.
- (c) Evaluate $f(x)$ at the endpoints.

Among these evaluations is our answer.

The biggest output = the absolute max and it occurs at the corresponding x value(s).

The smallest output = the absolute min and it occurs at the corresponding x value(s).

4.3: 1st and 2nd Derivatives and Graphs

1. We explored the following connections between 1st derivatives and the original function $f(x)$ for x in the domain of f :

$f'(x) > 0$ (derivative positive)	\Leftrightarrow	$f(x)$ is increasing
$f'(x) < 0$ (derivative negative)	\Leftrightarrow	$f(x)$ is decreasing
$f'(x) = 0$	\Leftrightarrow	$f(x)$ has a horizontal tangent
$f'(x)$ Does Not Exist	\Leftrightarrow	$f(x)$ has a vert. tangent, a sharp corner or not continuous
2. In particular, we used this to find the intervals of increase and decrease and to classify critical points as local max or local min or neither. We summarized this last fact in the following theorem:

The First Derivative Test:

If $x = c$ is a critical point of $f(x)$, then

- (a) If $f'(c)$ changes from negative to positive at $x = c$, then $x = c$ corresponds to a local minimum of f .
 - (b) If $f'(c)$ changes from positive to negative at $x = c$, then $x = c$ corresponds to a local maximum of f .
 - (c) If $f'(c)$ does not change at $x = c$, then $x = c$ is not a local max and not a local min of f .
3. We then explored the connections between 2nd derivatives and the original function $f(x)$ for x in the domain of f :

$f''(x) > 0$ (2nd deriv. positive)	\Leftrightarrow	$f(x)$ is concave up
$f''(x) < 0$ (2nd deriv. negative)	\Leftrightarrow	$f(x)$ is concave down
$f''(x) = 0$	\Leftrightarrow	inconclusive about concavity, <i>possible</i> inflection point
$f''(x)$ Does Not Exist	\Leftrightarrow	inconclusive about concavity, <i>possible</i> inflection point
 4. If we find that $f''(c) = 0$ or $f''(c)$ does not exist, we can determine if $x = c$ is an inflection point by checking the values of $f''(x)$ at points near $x = c$ and using the definition of inflection point: If $f''(x)$ changes from positive to negative (or from negative to positive) at $x = c$, then $x = c$ is an inflection point.

5. We also made the observation that if $x = c$ is a critical number and f is concave up, then $x = c$ is a local min. And if $x = c$ is a critical number and f is concave down, then $x = c$ is a local max. We called this the

The Second Derivative Test:

If $x = c$ is a critical point of $f(x)$, then

- (a) If $f''(c) > 0$, then $x = c$ corresponds to a local minimum of f .
- (b) If $f''(c) < 0$, then $x = c$ corresponds to a local maximum of f .
- (c) If $f''(c) = 0$ or if $f''(c)$ does not exist, then the test is inconclusive (we can't conclude if $x = c$ is a local max/min or neither based on this information alone).