

Week 8 Overview

My reviews and review sheets are not meant to be your only form of studying. It is vital to your success on the exams that you carefully go through and understand ALL the homework problems, worksheets and lecture material. Hopefully this review sheet will remind you of some of the key ideas of these sections.

4.4: L'Hopital's Rule

1. Here we learned a valuable tool for evaluate limits by using derivatives in a special way.

L'Hopital's Rule says: If either

- (a) $\lim_{x \rightarrow a} f(x) = 0$ and $\lim_{x \rightarrow a} g(x) = 0$, or
- (b) $\lim_{x \rightarrow a} f(x) = \pm\infty$ and $\lim_{x \rightarrow a} g(x) = \pm\infty$

$$\text{then } \lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}.$$

Of course the functions must be differentiable around $x = a$ for this to make sense.

2. Thus, if we have the **indeterminant form** $\frac{0}{0}$ or $\frac{\infty}{\infty}$ in a limit calculation, then we can separately differentiate the top and bottom and then evaluate the limit and we get the same answer.
3. We saw that sometimes L'Hopital's rule must be used more than once in a limit calculation, but **we must always stop at each step and verify that L'Hopital's rule still applies.**
4. We then discussed how to handle other indeterminate forms by transforming them into one of the forms above. Here are those other forms and what usually works in each case:
 - (a) Indeterminant form $0 \cdot \infty$: Rewrite one of the terms in the product as $f(x) = \frac{1}{1/f(x)}$. This will give a $0/0$ or ∞/∞ form and you can then use L'Hopital's Rule.
 - (b) Indeterminant form $\infty - \infty$: Combine into one fraction (by common denominator, rationalizing, identities, factoring). This will give a $0/0$ or ∞/∞ form and you can then use L'Hopital's Rule.
 - (c) Indeterminant forms 0^0 , 1^∞ , ∞^0 : Label the limit you want as L and take the natural log of both sides. Now you can bring down the exponent. The new limit you have will be one of the forms above which you can appropriately change and use L'Hopital's Rule. At the end of the limit calculation, you will have $\ln(L)$ which you will need to exponentiate to get L .

4.5: Curve Sketching

1. We discussed how we can use all the tools from this course to get a good sketch of the graph of a function without a graphing calculator.
2. Here are the steps I gave (The order of some steps aren't that important, so it doesn't really matter if my ordering is different than the book or different from the order you like to do the steps. What is important is that you get all the information you can.):
 - (a) **Domain and other quick observations:** Find the domain, see if you can quickly determine if the function is even or odd, see if you can quickly determine if the function is periodic (is it a function of trig functions).
 - (b) **Asymptotes:** Find if there are horizontal asymptotes ($\lim_{x \rightarrow \pm\infty} f(x)$). If there are vertical asymptotes, figure out what is happening on each side of the asymptote ($\lim_{x \rightarrow a^-} f(x)$ and $\lim_{x \rightarrow a^+} f(x)$).
 - (c) **Critical Numbers:** Find $f'(x)$ and determine all critical numbers.
 - (d) **Possible Inflection Points:** Find $f''(x)$ and determine all possible inflection points.
 - (e) **Inc/Dec and Concave Up/Down:** Use the first and second derivatives to determine when the function is inc/dec or concave up/down. I always summarize this information in a number line.
 - (f) **Compute Values:** Evaluate $f(x)$ at all critical points and possible points of inflection so that you can plot them. It usually nice to plot a few more points, namely the y -intercept (when $x = 0$, so $y = f(0)$). If *it is easy to do so*, find the x -intercepts as well (solve $f(x) = 0$).
 - (g) **Plot:** Draw asymptotes (indicate how the function approaches it). Plot all points you know from above. Connect dots according to the information you have about inc/dec and concave up/down.