Overview of 3.9: Related Rates

Idea: In a given scenario, it is sometimes common that the rate of change of one quantity is known (or can be found). If we know the relationships between the quantities themselves, then a related rates question asks to find the relationships between the rates of the quantities. In other words, once we find one rate in a problem, how can we find the others? Depending on the scenario, there certainly can be some rates that are constants while others that depend on the values of the quantities themselves. We shouldn't be surprised by this. The key to these problems is to first find the general relationships between the rates (*before plugging in any values*), then evaluate this relationship with the particular quantities in question.

Recipe for Solving a Related Rates Problem:

Step 1: Draw a good picture. Label **everything**and give variable names to any changing quantities.

Step 2: Determine what information you **know** and what you **want** to find (and where).

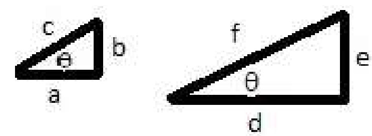
Step 3: Find an equation relating the relevant variables. This usually involves a formula from geometry (inside the front cover of your text), similar triangles, the Pythagorean Theorem, or a formula from trigonometry. Use your picture!

Step 4: Use implicit differentiation to differentiate the equation with respect to time t.

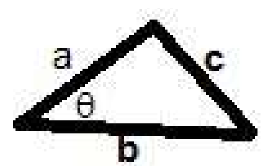
Step 5: Substitute in what you **know** from Step 2 into **all** the relationships you have found and solve for the quantity you **want**. Do **not** substitute before this step!

A few tools

- Area of a circle $= \pi r^2$, Circumference of a circle $= 2\pi r$.
- Volume of a sphere $= \frac{4}{3}\pi r^3$, Surface area of a sphere $= 4\pi r^2$
- Volume of a cone $= \frac{1}{3}\pi r^2 h$, Volume of a cylinder $= \pi r^2 h$.
- Similar Triangles: $\frac{a}{b} = \frac{d}{e}$ and $\frac{b}{c} = \frac{e}{f}$ and so on... for a situation such as:



- Trig Definitions: $\tan(\theta) = \frac{b}{a}$, $\sin(\theta) = \frac{b}{c}$, $\cos(\theta) = \frac{b}{c}$ for the right triangle pictured above.
- Pythagorean Theorem: $a^2 + b^2 = c^2$ for **right** triangles as in picture above.
- Law of Cosines: $a^2+b^2-2ab\cos(\theta) = c^2$ for all triangles as pictured here:



- Law of Sines: $\frac{b}{a} = \frac{\sin(B)}{\sin(A)}$ for all triangles as pictured here: