Quiz Soln's

Evaluate all of the following:

1. $\int \frac{y}{\sqrt{y^2-7}}\,dy$

$u = y^2 - 7$
$du = 2y\,dy$
$dy = \frac{du}{2y}$

$= \int \frac{y}{\sqrt{u}} \frac{du}{2y} = \frac{1}{2}\int u^{-1/2}\,du$

$= \frac{1}{2}\left(\frac{1}{1/2}u^{1/2}\right) + C$ $= (y^2-7)^{1/2} + C$

$= \sqrt{y^2-7} + C$

2. $\int_{\pi/6}^{\pi/3} \frac{\sin(3x)}{2+\cos(3x)}\,dx$

$u = 2 + \cos(3x)$
$du = 3\sin(3x)\,dx$
$dx = \frac{du}{3\sin(3x)}$
$x = \frac{\pi}{3} \Rightarrow u = 2 + \cos(\pi) = 1$
$x = \frac{\pi}{6} \Rightarrow u = 2 + \cos(\pi/3) = 2$

$= \int_1^2 \frac{\sin(3x)}{u} \frac{du}{3\sin(3x)}$

$= \frac{1}{3}\int_1^2 \frac{1}{u}\,du$

$= \frac{1}{3}\ln(u)\Big|_1^2 = \left(\frac{1}{3}\ln(2)\right) - \left(\frac{1}{3}\ln(1)\right) = \frac{1}{3}\ln(2)$

3. $\int \tan(x)\ln(\cos(x))\,dx$

$u = \ln(\cos(x))$
$du = \frac{-1}{\cos(x)}\sin(x)\,dx = -\tan(x)\,dx$
$dx = \frac{du}{-\tan(x)}$

$= \int \tan(x)\, u \frac{du}{-\tan(x)}$

$= -\int u\,du$

$= -\frac{1}{2}u^2 + C = -\frac{1}{2}(\ln(\cos(x)))^2 + C$

4. $\int \frac{y^3}{\sqrt{y^2-7}}\,dy$

$u = y^2 - 7 \quad y^2 = u + 7$
$du = 2y\,dy$
$dy = \frac{du}{2y}$

$= \int \frac{y^3}{\sqrt{u}} \frac{du}{2y} = \frac{1}{2}\int \frac{y^2}{u^{1/2}}\,du$

$= \frac{1}{2}\int \frac{u+7}{u^{1/2}}\,du = \frac{1}{2}\int u^{1/2} + 7u^{-1/2}\,du$

$= \frac{1}{2}\left[\frac{2}{3}u^{3/2} + 7\frac{1}{1/2}u^{1/2}\right] + C = \frac{1}{3}(y^2-7)^{3/2} + 7(y^2-7)^{1/2} + C$