1. (13 pts) Evaluate the integrals. If you do a substitution in a definite integral problem, you must show me that you can appropriately change the bounds to get full credit. Simplify your final answers.

(a) $\displaystyle\int_0^{\pi/6} \frac{\sin(2x)}{(\cos(2x))^4}\,dx$

$u = \cos(2x)$

$du = -2\sin(2x)\,dx$

$\frac{1}{-2\sin(2x)}\,du = dx$

$\displaystyle\int_1^{1/2} \frac{\cancel{\sin(2x)}}{u^4}\,\frac{1}{-2\cancel{\sin(2x)}}\,du$

$\displaystyle\frac{1}{2}\int_{1/2}^{1} u^{-4}\,du$

$\displaystyle\frac{1}{2}\left[\frac{1}{-3}u^{-3}\Big|_{1/2}^{1}\right] = -\frac{1}{6}\left[\frac{1}{1^3} - \frac{1}{(1/2)^3}\right]$

$= -\frac{1}{6}[1 - 8] = \boxed{\frac{7}{6}}$

(b) $\displaystyle\int x^3\sqrt{x^2+5}\,dx$

$u = x^2 + 5 \Leftrightarrow x^2 = u - 5$

$du = 2x\,dx$

$\frac{1}{2x}\,du = dx$

$\displaystyle\int x^3 \sqrt{u}\,\frac{1}{2x}\,du$

$\displaystyle= \frac{1}{2}\int (u-5)\sqrt{u}\,du$

$\displaystyle= \frac{1}{2}\int u^{3/2} - 5u^{1/2}\,du$

$\displaystyle= \frac{1}{2}\left[\frac{2}{5}u^{5/2} - \frac{10}{3}u^{3/2}\right] + C$

$\displaystyle= \boxed{\frac{1}{5}(x^2+5)^{5/2} - \frac{5}{3}(x^2+5)^{3/2} + C}$

2. (12 pts)

(a) Evaluate $\int_0^3 |6x^2 + 6x - 12|\, dx$

$$6x^2+6x-12 = 6(x^2+x-2) = 6(x+2)(x-1) \stackrel{?}{=} 0$$

$$x=-2 \text{ or } x=1$$

$$\int_0^1 6x^2+6x-12\,dx = 2x^3+3x^2-12x \Big|_0^1 = 2+3-12 = -7$$

$$\int_1^3 6x^2+6x-12\,dx = 2x^3+3x^2-12x \Big|_1^3$$

$$= \left(2(3)^3+3(3)^2-12(3)\right) - (-7) = 54+27-36+7$$

$$= 52$$

$$\int_0^3 |6x^2+6x-12|\,dx = 7+52 = \boxed{59}$$

(b) Let $g(x) = \int_{2x^2}^{10} \sin(\pi t^2)\, dt$. Compute $g'(1/2)$.

$$g'(x) = -\sin(\pi 4x^4)\cdot 4x$$

$$\Rightarrow g'\left(\tfrac{1}{2}\right) = -\sin\left(4\pi\left(\tfrac{1}{2}\right)^4\right)\cdot 4\left(\tfrac{1}{2}\right)$$

$$= -2\sin\left(\pi/4\right) = -2\,\tfrac{\sqrt{2}}{2} = \boxed{-\sqrt{2}}$$

3. (11 pts) (The two problems below are NOT related)

(a) If $\int_0^4 f'(x)\,dx = 10$, $\int_3^4 f'(x)\,dx = 2$, and $f(3) = 13$, then what is the value of $f(0)$?

$$\underbrace{\int_0^4 f'(x)\,dx}_{10} - \underbrace{\int_3^4 f'(x)\,dx}_{2} = \int_0^3 f'(x)\,dx = \underbrace{f(3)}_{} - f(0)$$

$$10 - 2 = \int_0^3 f'(x)\,dx = 13 - f(0)$$

$$\Rightarrow \quad 8 = 13 - f(0) \quad \Rightarrow \quad \boxed{f(0) = 5}$$

(b) A tomato is thrown downward from the top of a tall building. At $t = 3$ seconds after being thrown, the tomato is at a height of 240 feet and is traveling at a *downward* velocity of 110 feet/sec. Assume the acceleration of the tomato due to gravity is $a(t) = -32$ ft/sec^2. Find the height of building.

$$a(t) = -32$$

$$v(t) = -32t + C$$

$$h(t) = -16t^2 + Ct + D$$

$$v(3) = -110 \Rightarrow -32(3) + C = -110$$

$$\Rightarrow C = -14$$

$$h(3) = 240 \Rightarrow -16(3)^2 - 14(3) + D = 240$$

$$\Rightarrow D = 240 + 16 \cdot 9 + 42$$

$$= \boxed{426 \text{ feet}}$$

4. (12 pts) (The two problems below are NOT related)

(a) Consider the region bounded by $y = e^x$, $y = 0$, $x = 0$ and $x = 2$. Find the value of a such that the vertical line $x = a$ divides this region into two subregions of equal area (*i.e.* find the vertical line $x = a$ that *bisects* the area).

$$\int_0^2 e^x\,dx = e^x\Big|_0^2 = e^2 - 1$$

WANT

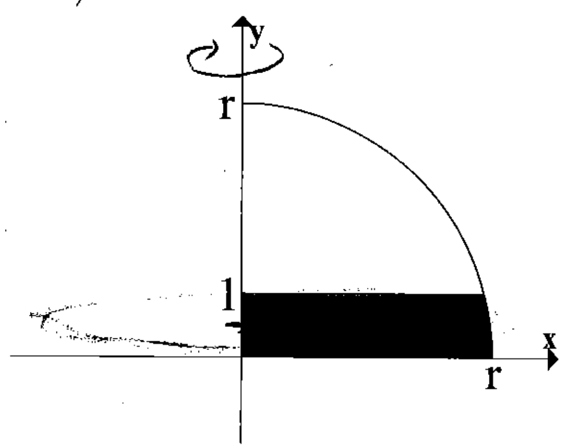
$$\int_0^a e^x\,dx = \frac{1}{2}(e^2 - 1)$$

$$\Rightarrow e^{\cancel{x}}\Big|_0^a = e^a - 1 = \frac{1}{2}(e^2 - 1) = \frac{1}{2}e^2 - \frac{1}{2}$$

$$\Rightarrow e^a = \frac{1}{2}e^2 + \frac{1}{2} \Rightarrow \boxed{a = \ln\left(\frac{1}{2}e^2 + \frac{1}{2}\right)}$$

≈ 1.4338

(b) Suppose r is a number bigger than 1. Let A be the region in the first quadrant that is below $y = 1$ and inside the circle $x^2 + y^2 = r^2$ (shown below). Find the volume of the solid obtained by rotating A about the y-axis. (Answer will involve r).



$$\int_0^1 \pi\left(\sqrt{r^2 - y^2}\right)^2 dy$$

$$\pi \int_0^1 r^2 - y^2\,dy$$

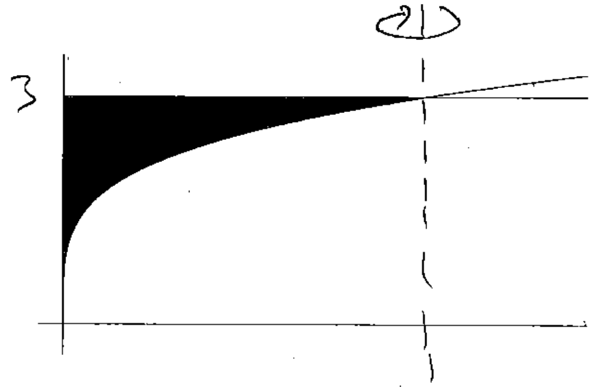
$$\pi\left(r^2 y - \frac{1}{3}y^3\Big|_0^1\right)$$

$$= \boxed{\pi\left(r^2 - \frac{1}{3}\right)}$$

Disc!

5. (12 pts) Let R be the region bounded by $y = 3$, $x = 0$ and $y = 3\sqrt[4]{x}$ (shown below)

(a) Find the area of this region.



$$\int_0^1 3 - 3x^{1/4}\,dx$$

or

$$= \int_0^3 \frac{1}{81} y^4\,dy$$

$$= \frac{1}{81}\,\frac{1}{5}\,y^5 \Big|_0^3$$

$$= \frac{1}{81}\,\frac{1}{5}\,(3)^5 = \boxed{3/5} = 0.6$$

(b) A solid is obtained by rotating the region R around the **vertical** line $x = 1$. Set up the integrals for the volume of this solid using BOTH the method of cylindrical shells and the method of washers (DO NOT EVALUATE).

Shells: $\boxed{\int_0^1 2\pi(1-x)(3-3x^{1/4})\,dx}$

Washers: $\boxed{\int_0^3 \pi(1)^2 - \pi\left(1 - \frac{1}{81}y^4\right)^2 dy}$

$$= \frac{13\pi}{15} \approx 2.7227$$