MATH 125 $\boxed{\text{EXAM 2}}$ Fall 2009

① (a) $\int \frac{2x+1}{x^2+3x-10}\,dx = \int \frac{2x+1}{(x+5)(x-2)}\,dx$

$\frac{2x+1}{(x+5)(x-2)} = \frac{A}{x+5} + \frac{B}{x-2}$

$2x+1 = A(x-2) + B(x+5)$

$\begin{cases} x=2 \Rightarrow 5 = 7B \Rightarrow B = 5/7 \\ x=-5 \Rightarrow -9 = -7A \Rightarrow A = 9/7 \end{cases}$

$\int \frac{9/7}{x+5} + \frac{5/7}{x-2}\,dx = \boxed{\frac{9}{7}\ln|x+5| + \frac{5}{7}\ln|x-2| + C}$

(b) $\int_1^9 \sqrt{x}\ln(x)\,dx$

$u = \ln(x)$ $dv = \sqrt{x}\,dx$

$du = \frac{1}{x}\,dx$ $v = \frac{2}{3}x^{3/2}$

$= \frac{2}{3}x^{3/2}\ln(x)\Big|_1^9 - \int_1^9 \frac{2}{3}x^{1/2}\,dx$

$= \left[\frac{2}{3}9^{3/2}\ln(9) - \frac{2}{3}1^{3/2}\ln(1)\right] - \frac{4}{9}x^{3/2}\Big|_1^9$

$= \left[\frac{2}{3}\,27\ln(9) - 0\right] - \left[\frac{4}{9}(9)^{3/2} - \frac{4}{9}(1)^{3/2}\right]$

ALL ACCEPTABLE

$= 18\ln(9) - \frac{4}{9}27 + \frac{4}{9}$

$= 18\ln(9) - \frac{108}{9} + \frac{4}{9} = 18\ln(9) - \frac{104}{9} = 18\ln(9) - 11.\overline{5}$

$= 36\ln(3) - \frac{104}{9} = 36\ln(3) - 11.\overline{5}$

≈ 27.9944868365

---

② (a) $\int \frac{\sin^2(x)\tan(x)}{\sec(x)}\,dx$

$\left\{ \frac{\tan(x)}{\sec(x)} = \frac{\sin(x)/\cos(x)}{1/\cos(x)} = \sin(x). \right.$

$= \int \sin^3(x)\,dx$

$= \int \sin^2(x)\sin(x)\,dx$

$= \int (1-\cos^2(x))\sin(x)\,dx$

$u = \cos(x)$

$du = -\sin(x)\,dx$

$dx = \frac{1}{-\sin(x)}\,du$

$= -\int 1-u^2\,du$

$= -\left[u - \frac{1}{3}u^3\right] + C$

$= \frac{1}{3}u^3 - u + C$

$= \boxed{\frac{1}{3}\cos^3(x) - \cos(x) + C}$

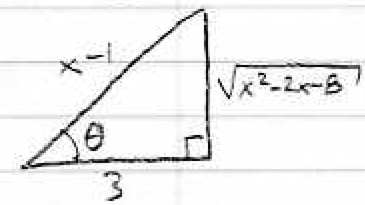
(b) $\displaystyle\int \frac{x}{\sqrt{x^2-2x-8}}\,dx = \int \frac{x}{\sqrt{(x-1)^2-9}}\,dx$

$x^2-2x-8 = x^2-2x+1-1-8 = (x-1)^2-9$

$x-1 = 3\sec(\theta) \qquad 0 \le \theta < \pi/2$

$dx = 3\sec(\theta)\tan(\theta)\,d\theta$

$\displaystyle\int \frac{(3\sec(\theta)+1)}{\sqrt{9\sec^2(\theta)-9}}\, 3\sec(\theta)\tan(\theta)\,d\theta \qquad \sec(\theta) = \frac{x-1}{3}$



$\displaystyle\int \frac{3\sec(\theta)+1}{\cancel{3}\cancel{\tan(\theta)}}\, \cancel{3}\sec(\theta)\cancel{\tan(\theta)}\,d\theta$

$= \displaystyle\int 3\sec^2(\theta) + \sec(\theta)\,d\theta$

$= 3\tan(\theta) + \ln|\sec(\theta)+\tan(\theta)| + C$

$= \cancel{3}\,\dfrac{\sqrt{x^2-2x-8}}{\cancel{3}} + \ln\left|\dfrac{x-1}{3} + \dfrac{\sqrt{x^2-2x-8}}{3}\right| + C$

$\boxed{\begin{aligned} &= \sqrt{x^2-2x-8} + \ln\left|\frac{x-1}{3} + \frac{\sqrt{x^2-2x-8}}{3}\right| + C \\ &= \sqrt{x^2-2x-8} + \ln\left|x-1+\sqrt{x^2-2x-8}\right| + C_1 \end{aligned}}$ } BOTH CORRECT

3 (a) $\displaystyle\int \frac{\ln(x)\cos(3\ln(x))}{x}\,dx$

$t = \ln(x)$

$dt = \frac{1}{x}\,dx$

$dx = x\,dt$

$\displaystyle\int t\cos(3t)\,dt$

$= \frac{1}{3}t\sin(3t) - \displaystyle\int \frac{1}{3}\sin(3t)\,dt \qquad u = t \qquad dv = \cos(3t)\,dt$

$= \frac{1}{3}t\sin(3t) + \frac{1}{9}\cos(3t) + C \qquad du = dt \qquad v = \frac{1}{3}\sin(3t)$

$= \boxed{\frac{1}{3}\ln(x)\sin(3\ln(x)) + \frac{1}{9}\cos(3\ln(x)) + C}$

(b) (i) $\displaystyle\int_1^4 x\sqrt{x^2-1}\,dx \qquad u = x^2-1 \qquad x=1 \Rightarrow u=0$

$du = 2x\,dx \qquad x=4 \Rightarrow u=15$

$dx = \frac{1}{2x}\,du$

$\displaystyle\int_0^{15} \cancel{x}\sqrt{u}\,\frac{1}{2\cancel{x}}\,du$

$\frac{1}{2}\cdot\frac{2}{3}u^{3/2}\Big|_0^{15} = \frac{1}{3}15^{3/2} - \frac{1}{3}0^{3/2} = 15^{3/2}/3 \approx 19.364916731$

(ii) $\Delta x = \frac{4-1}{4} = 3/4 \qquad x_0 = 1,\ x_1 = 1.75,\ x_2 = 2.5,\ x_3 = 3.25,\ x_4 = 4$

$S_4 = \frac{1}{3}\Delta x\,[f(1) + 4f(1.75) + 2f(2.5) + 4f(3.25) + f(4)]$

$= \frac{1}{3}\cdot\frac{3}{4}\big[1\sqrt{1^2-1} + 4(1.75)\sqrt{1.75^2-1} + 2(2.5)\sqrt{2.5^2-1}$

$+ 4(3.25)\sqrt{3.25^2-1} + 4\sqrt{4^2-1}\big]$

≈ 19.3004092759 ONE DIGIT!

4 (a) $\frac{1}{b-a}\int_a^b f(x)\,dx$

either substitute $u = x+1$ or divide

$\frac{1}{10-4}\int_4^{10} \frac{100x}{x+1}\,dx$

$$x+1 \overline{\big)\,100x\,}\; 100$$
$$-(100x+100)$$
$$-100$$

$\frac{1}{6}\int_4^{10} 100 - \frac{100}{x+1}\,dx$

$\frac{1}{6}\left[100x - 100\ln|x+1|\,\Big|_4^{10}\right]$

$\frac{1}{6}\left[(1000 - 100\ln(11)) - (400 - 100\ln(5))\right]$

$\frac{1}{6}\left[600 - 100\ln(11) + 100\ln(5)\right]$ ← ALL ACCEPTABLE

$$\boxed{\begin{aligned} &= 100 - \tfrac{100}{6}\ln(11) + \tfrac{100}{6}\ln(5) \\ &= 100 + \tfrac{50}{3}\ln(5/11) \approx 86.8590439939 \end{aligned}}$$

(b) $86.8590439939 = \frac{100x}{x+1}$

$86.8590439939x + 86.8590439939 = 100x$

$86.8590439939 = 13.1409560061x$

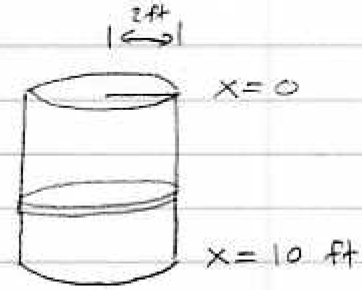
$\boxed{x \approx 6.6097964225 \text{ hours}}$

EXACT: $100 + \frac{50}{3}\ln(5/11) = \left(100 - 100 - \frac{50}{3}\ln(5/11)\right)x$

$x = \frac{100 + 50/3\ln(5/11)}{-50/3\ln(5/11)} =$

$\boxed{x \approx 6.61 \text{ hours}}$

(5) (a) Let x = dist. from top.



Force of a horiz. slice $= 62.5(\text{Area})\,\Delta x$

$= 62.5\,\pi r^2\,\Delta x$

$= 62.5\pi\, 2^2\,\Delta x$

Dist $= x$

$$\text{WORK} = \underset{\text{HALF FULL}}{\int_5^{10}}\ \underset{\text{DIST}}{x}\ \underbrace{62.5\pi\, 2^2\, dx}_{\text{FORCE}}$$

$= 250\pi \int_5^{10} x\,dx$

$= 250\pi\ \frac{1}{2}x^2\Big|_5^{10} = 125\pi\left[10^2 - 5^2\right]$

$= 125\pi(75) = \boxed{9375\pi \text{ ft-lbs}}$

$\approx \boxed{29452.4311274 \text{ ft-lbs}}$

(b)

$\text{WORK} = \int_0^8 F(x)\,dx$

$= \int_0^8 40 + 50e^{-x/2}\,dx$

$= 40x + 50(-2)e^{-x/2}\Big|_0^8$

$= \left(40(8) - 100e^{-8/2}\right) - \left(40(0) - 100e^{-0/2}\right)$

$= 320 - 100e^{-4} + 100$

$= 420 - 100e^{-4}$ Joules

$\approx 420 - \frac{100}{e^4}$

$\approx \boxed{418.1684361 \text{ Joules}}$