1. Evaluate the integrals.

(a) (5 pts) $\displaystyle\int \frac{2\sqrt{x} - 6x^2 + 3x^4}{2x^4}\, dx$

$$= \int \frac{2x^{1/2}}{2x^4} - \frac{6x^2}{2x^4} + \frac{3x^4}{2x^4}\, dx$$

$$= \int x^{-7/2} - 3x^{-2} + \frac{3}{2}\, dx$$

$$\boxed{\begin{aligned} &= -\frac{2}{5}x^{-5/2} - \frac{3}{-1}x^{-1} + \frac{3}{2}x + C \\ &= -\frac{2}{5}x^{-5/2} + \frac{3}{x} + \frac{3}{2}x + C \end{aligned}}$$

(b) (6 pts) $\displaystyle\int_{\pi/6}^{\pi/2} \frac{\cos(x)}{3 - 2\sin(x)}\, dx$

$u = 3 - 2\sin(x)$

$du = -2\cos(x)dx$

$dx = \frac{1}{-2\cos(x)} du$

$$= \int_2^1 \frac{\cos(x)}{u} \frac{1}{-2\cos(x)}\, du$$

$$= \frac{1}{2}\int_1^2 \frac{1}{u}\, du$$

$$= \frac{1}{2}\ln|u|\Big|_1^2$$

$$= \frac{1}{2}\ln|2| - \frac{1}{2}\ln(1)$$

$$= \boxed{\frac{1}{2}\ln(2)}$$

2. Evaluate the integrals.

(a) (5 pts) $\int x\sqrt{x-2}\,dx$

$u = x - 2 \qquad x = u + 2$
$du = dx$

$\int (u+2)\sqrt{u}\,du$

$= \int u^{3/2} + 2u^{1/2}\,du$

$= \frac{2}{5}u^{5/2} + 2\frac{2}{3}u^{3/2} + C$

$= \boxed{\frac{2}{5}(x-2)^{5/2} + \frac{4}{3}(x-2)^{3/2} + C}$

(b) (6 pts) $\int_{-1}^{0} \frac{x^3}{x^8+1}\,dx$ (Hint: What function has a derivative that looks like x^3?)

$u = x^4$
$du = 4x^3\,dx$
$dx = \frac{1}{4x^3}du$

$= \int_1^0 \frac{\cancel{x^3}}{u^2+1}\frac{1}{4\cancel{x^3}}\,du$

$= \frac{1}{4}\int_1^0 \frac{1}{u^2+1}\,du$

$= \frac{1}{4}\tan^{-1}(u)\Big|_1^0$

$= \frac{1}{4}\tan^{-1}(0) - \frac{1}{4}\tan^{-1}(1)$

$= 0 - \frac{1}{4}\pi/4 = \boxed{-\frac{\pi}{16}}$

3. The two parts below are not related.

(a) (6 pts) Your physics instructor gives you a function, $f(x)$, defined by $f(x) = \int_{-3x}^{x^2} e^{t^2}\, dt$. Find the slope of the tangent line to $f(x)$ at $x = 1$.

$$f(x) = \int_{-3x}^{0} e^{t^2} dt + \int_0^{x^2} e^{t^2} dt$$

$$f(x) = -\int_0^{-3x} e^{t^2} dt + \int_0^{x^2} e^{t^2} dt$$

$$f'(x) = -e^{(-3x)^2} \cdot (-3) + e^{(x^2)^2} \cdot 2x$$

$$\boxed{f'(1) = 3e^9 + 2e}$$

(b) (6 pts) A particle is traveling up and down along a straight line with velocity given by $v(t) = 4t^3 - 4t$ ft/sec at time t seconds.
Find the **total distance** traveled by the particle from $t = 0$ to $t = 2$.

$$\int_0^2 |4t^3 - 4t|\, dt = ?$$

$$4t^3 - 4t = 0 \Rightarrow 4t(t^2 - 1) = 0 \Rightarrow 4t(t+1)(t-1) = 0$$

$t = 0$, $t = -1$, or $t = 1$.

$$\int_0^1 4t^3 - 4t\, dt = t^4 - 2t^2 \Big|_0^1 = ((1)^4 - 2(1)^2) - 0 = -1$$

$$\int_1^2 4t^2 - 4t\, dt = t^4 - 2t^2 \Big|_1^2 = ((2)^4 - 2(2)^2) - ((1)^4 - 2(1)^2)$$

$$= 16 - 8 - -1 = 9$$

$$\int_0^2 |4t^3 - 4t|\, dt = 1 + 9 = \boxed{10}$$

4. The two parts below are not related.

(a) (6 pts) Use the Midpoint Rule with $n = 3$ subdivisions to approximate the value of the integral $\int_2^8 \sqrt{1+x^2}\,dx$. (Leave your answer expanded out, you don't need to simplify).

$$\Delta x = \frac{8-2}{3} = \frac{6}{3} = 2$$

$$x_0 = 2,\ x_1 = 4,\ x_2 = 6,\ x_3 = 8$$

$$\bar{x}_1 = 3,\ \bar{x}_2 = 5,\ \bar{x}_3 = 7$$

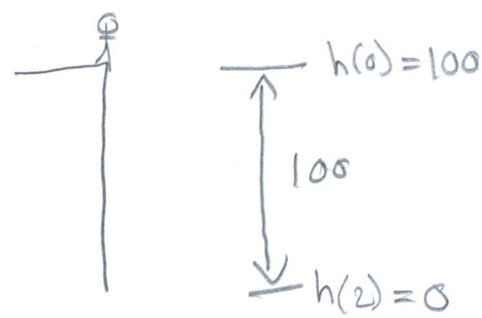
$$\boxed{\begin{aligned} &\sqrt{1+(3)^2}\cdot 2 + \sqrt{1+(5)^2}\cdot 2 + \sqrt{1+(7)^2}\cdot 2 \\ &= 2\sqrt{10} + 2\sqrt{26} + 2\sqrt{50} \end{aligned}}$$

(b) (6 pts) You are standing on the edge of a building 100 feet above a path (your instructor happens to be walking on the path). At what initial downward velocity would you have to 'accidentally' throw a water balloon in order for it to hit the path in 2 seconds? Assume acceleration is a constant 32 feet/second downward.

$$a(t) = -32$$

$$v(t) = -32t + C$$

$$h(t) = -16t^2 + Ct + D$$



$$h(0) = 100 \Rightarrow D = 100$$

$$h(2) = 0 \Rightarrow -16(2)^2 + C(2) + 100 = 0$$

$$-64 + 2C + 100 = 0$$

$$2C = -36$$

$$C = -18$$

$$\boxed{v(0) = -18 \text{ ft/sec}} \qquad \boxed{18 \text{ ft/sec downward}}$$

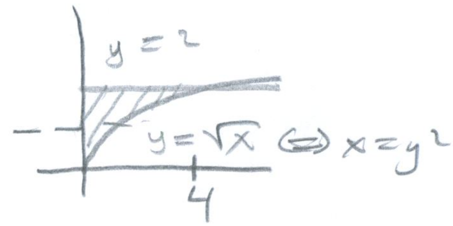
5. Consider the region, R, bounded between $y = \sqrt{x}$, the **horizontal** line $y = 2$, and the y-axis.

(a) (6 pts) Find a value of b, such that the **horizontal** line $y = b$ divides the region, R, into two regions of equal area.

SAME

$$\text{TOTAL AREA} = \int_0^2 y^2\,dy = \int_0^4 2-\sqrt{x}\,dx$$

$$= \frac{1}{3}y^3\Big|_0^2 = 8/3$$



$$\text{HALF} = \int_0^b y^2\,dy$$

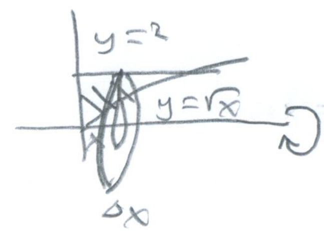
$$= \frac{1}{3}y^3\Big|_0^b = \frac{1}{3}b^3 \overset{?}{=} \frac{1}{2}\cdot\frac{8}{3}$$

$$b^3 = 4$$

$$\boxed{b = 4^{1/3} = \sqrt[3]{4}}$$

(b) (4 pts) Use cross-sectional slicing (*i.e.* discs/washers) to set up an integral that represents the volume of the solid obtained by rotating the region R about the x-axis. (DO NOT EVALUATE)

$$\boxed{\int_0^4 \pi(2)^2 - \pi(\sqrt{x})^2\,dx = \pi\int_0^4 4 - x\,dx}$$



(c) (4 pts) Use the method of cylindrical shells to set up an integral that represents the volume of the solid obtained by rotating the region R about the **horizontal** line $y = 5$. (DO NOT EVALUATE)

$$\boxed{\int_0^2 2\pi(5-y)\,y^2\,dy = 2\pi\int_0^2 5y^2 - y^3\,dy}$$