1. (12 points) Evaluate the integrals:

(a) $\int x^3\left(\frac{1}{x^4}-\frac{5}{\sqrt{x^7}}\right)\,dx = \int \frac{x^3}{x^4} - \frac{5x^3}{x^{7/2}}\,dx$

SIMPLIFY

$= \int \frac{1}{x} - 5x^{-1/2}\,dx$

$= \boxed{\ln|x| - \frac{5}{1/2}x^{1/2} + C}$

$= \boxed{\ln|x| - 10\sqrt{x} + C}$

(b) $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = \int_1^2 \frac{e^u}{\cancel{\sqrt{x}}}\,2\cancel{\sqrt{x}}\,du$

$= 2\int_1^2 e^u\,du$

$= 2\left(e^u\Big|_1^2\right)$

$= \boxed{2(e^2-e)}$

$u=\sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$dx = 2\sqrt{x}\,du$

$x=1 \Rightarrow u=1$

$x=4 \Rightarrow u=2$

(c) $\int \frac{x^3}{(1+x^2)^5}\,dx = \int \frac{x^{\cancel{3}\,2}}{u^5}\,\frac{1}{2\cancel{x}}\,du$

$= \frac{1}{2}\int \frac{u-1}{u^5}\,du$

$= \frac{1}{2}\int \frac{1}{u^4} - \frac{1}{u^5}\,du = \frac{1}{2}\int u^{-4} - u^{-5}\,du$

$= \frac{1}{2}\left(\frac{1}{-3}u^{-3} - \frac{1}{-4}u^{-4}\right) + C$

$= \boxed{-\frac{1}{6}(1+x^2)^{-3} + \frac{1}{8}(1+x^2)^{-4} + C}$

$u = 1+x^2 \Rightarrow x^2 = u-1$

$du = 2x\,dx$

$dx = \frac{1}{2x}\,du$

2. (6 pts) If $f(x) = \int_{\sin(5x)}^{2} e^t \sqrt{t+3}\, dt$, find the derivative of $f(x)$ and evaluate it at $x = \pi$.
That is, find the value of $f'(\pi)$.

FLIPPING BOUNDS $\rightarrow$ $f(x) = -\int_{2}^{\sin(5x)} e^{t} \sqrt{t+3}\, dt$

FTOC (Part I) $\rightarrow$ $f'(x) = -e^{\sin(5x)} \sqrt{\sin(5x)+3}\; 5\cos(5x)$

$$f'(\pi) = -e^{\overset{0}{\sin(5\pi)}} \sqrt{\underbrace{\sin(5\pi)}_{0}+3}\; 5\underbrace{\cos(5\pi)}_{-1}$$

$$\boxed{f'(\pi) = 5\sqrt{3}}$$

3. (10 pts) A particle is moving on a straight line with acceleration given by $a(t) = 6t$, where t is in seconds. At $t = 2$ seconds, you measure that the velocity of the particle is $v(2) = -15$.

(a) Find the velocity function, $v(t)$, for the particle at time t.

$a(t) = 6t$

$v(t) = 3t^2 + C$

$v(2) = -15$

$3(2)^2 + C = -15$

$C = -27$

$$\boxed{v(t) = 3t^2 - 27}$$

(b) Find the **total distance** traveled by the particle from $t = 0$ to $t = 5$.

$\int_0^5 |3t^2 - 27|\, dt$

ZEROS: $3t^2 - 27 = 0$

$t^2 = 9$

$t = \pm 3$

$\int_0^3 3t^2 - 27\, dt = t^3 - 27t \Big|_0^3 = [(3)^3 - 27(3)] - [(0)^3 - 27(0)]$
$= -54$

$\int_3^5 3t^2 - 27\, dt = t^3 - 27t \Big|_3^5 = [(5)^3 - 27(5)] - [(3)^3 - 27(3)]$
$= -10 - -54 = 44$

$\int_0^5 |3t^2 - 27|\, dt = 54 + 44 = \boxed{98}$

4. Consider

$$\int_1^7 (x^2+1)^{1/3}\,dx$$

(a) (6 pts) Use the left-endpoint rule with $n = 4$ rectangles to approximate the value of this definite integral. Show you work, then give your final answer rounded to 3 digits after the decimal.

$$\Delta x = \frac{7-1}{4} = \frac{3}{2} = 1.5$$

$x_0 = 1,\ x_1 = 2.5,\ x_2 = 4,\ x_3 = 5.5,\ x_4 = 7$

$$L_4 = \Big[\underbrace{(1^2+1)^{1/3}}_{f(x_0)} + \underbrace{(2.5^2+1)^{1/3}}_{f(x_1)} + \underbrace{(4^2+1)^{1/3}}_{f(x_2)} + \underbrace{(5.5^2+1)^{1/3}}_{f(x_3)}\Big]\underbrace{1.5}_{\Delta x}$$

$$= [1.25992105 + 1.93543832 + 2.571281591 + 3.149802625]\cdot 1.5$$

$$= 8.916443586 \cdot 1.5 = 13.37466538$$

$$\boxed{13.375}$$

(b) (2 pt) Is your answer an overestimate or underestimate? (You must explain to get full credit)

$y = (x^2+1)^{1/3}$ is an increasing function ← BECAUSE $y' = \underbrace{\frac{1}{3}(x^2+1)^{-2/3}\,2x}_{\text{positive for } x \text{ between 1 and 7}}$

So L_4 is an $\boxed{\text{underestimate}}$

ASIDE: ACTUAL VALUE ≈ 15.2092

5. (8 pts) Find the area of the region bounded by $y = x^3$ and $32x = y^2$.

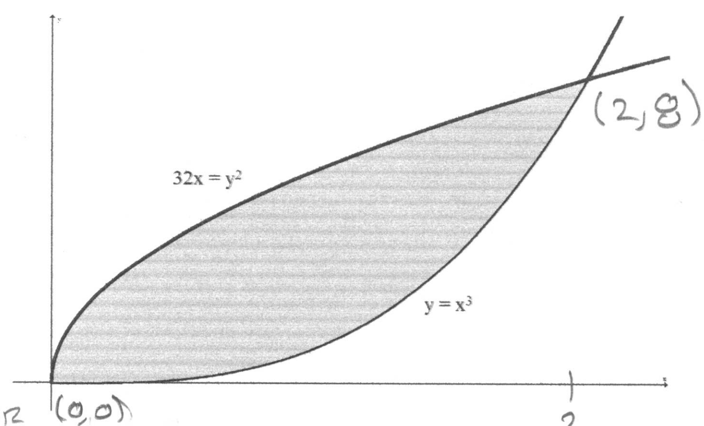
$\boxed{\text{INTERSECTION}}$

① $32x = y^2$ ② $y = x^3$

① & ② ⇒ $32x = (x^3)^2$

$32x = x^6 \Rightarrow \begin{cases} x = 0 \text{ or} \\ 32 = x^5 \\ x = 2 \end{cases}$

$y = x^3 = 2^3 = 8$



$$\text{AREA} = \int_0^2 \sqrt{32x} - x^3\,dx \overset{\text{OR}}{=} \int_0^8 y^{1/3} - \frac{1}{32}y^2\,dy$$

$$= \sqrt{32}\,\frac{2}{3}x^{3/2} - \frac{1}{4}x^4\Big|_0^2 \qquad = \frac{3}{4}y^{4/3} - \frac{1}{96}y^3\Big|_0^8$$

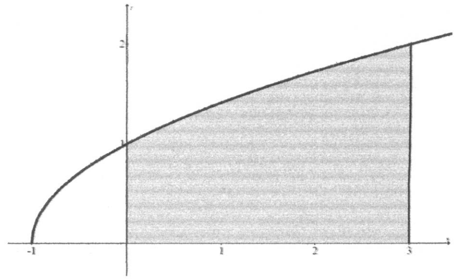
$$= \Big[\frac{2}{3}\sqrt{32}\,2^{3/2} - \frac{1}{4}2^4\Big] - 0 \qquad = \Big[\frac{3}{4}8^{4/3} - \frac{1}{96}8^3\Big] - 0$$

$$= \frac{2}{3}\sqrt{32}\,2\sqrt{2} - 4 = \frac{32}{3} - 4 = \boxed{\frac{20}{3}} \qquad = \frac{3}{4}2^4 - \frac{1}{96}512 = 12 - \frac{16}{3} = \boxed{\frac{20}{3}}$$

$$= 6.\overline{6}$$

6. (16 points)

Consider the region, R, bounded by the curve $y = \sqrt{x+1}$, the x-axis, and between $x = 0$ and $x = 3$. A picture of this region is given at right.



(a) (4 pts) Set up an integral (DO NOT EVALUATE) for the volume of the solid obtained by rotating the region R about the **horizontal line** $y = -3$.

WITH CROSS-SECTIONAL SLICING: $\boxed{\int_0^3 \pi\left(\sqrt{x+1}+3\right)^2 - \pi(3)^2\,dx}$

EITHER

WITH SHELLS: $\int_0^1 2\pi(y+3)\,3\,dy + \int_1^2 2\pi(y+3)\left(3-(y^2-1)\right)dy$

(b) (6 pts) Find the volume of the solid obtained by rotating the region R about the x-axis. Set up the integral AND evaluate.

PERPENDICULAR SLICING: $\int_0^3 \pi\left(\sqrt{x+1}\right)^2 dx = \pi\int_0^3 x+1\,dx$

$= \pi\left(\frac{1}{2}x^2 + x \Big|_0^3\right)$

$= \pi\left(\left(\frac{1}{2}(3)^2+3\right) - (0)\right)$

$= \pi\left(\frac{9}{2}+3\right)$

$= \boxed{\frac{15\pi}{2}} \approx 23.5619$

(c) (6 pts) Find the volume of the solid obtained by rotating the region R about the y-axis. Set up the integral AND evaluate.

SHELLS: $\int_0^3 2\pi x\sqrt{x+1}\,dx$

$u = x+1 \Rightarrow u-1 = x$
$du = dx$

$x=0 \Rightarrow u=1$
$x=3 \Rightarrow u=4$

$\int_1^4 2\pi(u-1)\sqrt{u}\,du$

$2\pi\int_1^4 u^{3/2} - u^{1/2}\,du$

$2\pi\left(\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\Big|_1^4\right)$

$2\pi\left[\left(\frac{2}{5}(4)^{5/2} - \frac{2}{3}(4)^{3/2}\right) - \left(\frac{2}{5} - \frac{2}{3}\right)\right]$

$2\pi\left[\frac{6\cdot 32}{15} - \frac{10\cdot 8}{15} - \frac{6}{15} + \frac{10}{15}\right] = 2\pi\left[\frac{116}{15}\right] = \boxed{\frac{232\pi}{15}}$

≈ 48.59966