Math 125H - Winter 2011
Exam 2
February 24, 2011

Name: _____

Section: _____

Student ID Number: _____

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- There are 5 pages of questions. Make sure your exam contains all these pages.
- You are allowed to use a scientific calculator (**no graphing calculators**) and one **hand-written** 8.5 by 11 inch page of notes.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.** Give exact answers wherever possible.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 80 minutes to complete the exam. Budget your time wisely.
  SPEND NO MORE THAN 15 MINUTES PER PAGE!

GOOD LUCK!

1. (12 points) Compute the following integrals.

(a) $\displaystyle\int_1^4 \frac{y \ln(\sqrt{y})}{\sqrt{y}}\, dy$

(b) $\displaystyle\int \frac{1}{(x^2 - b^2)^{3/2}}\, dx$. ($b$ is a positive constant).

2. (12 points) Compute the following integrals.

(a) $\displaystyle\int \frac{7x+3}{(x-1)(x^2+1)}\,dx.$

(b) $\displaystyle\int \frac{x}{\sqrt{x^2+8x+25}}\,dx.$

3. (12 points)

(a) Using integration by parts and bit of algebraic manipulation, show that for any positive integer $n \geq 2$,

$$\int_0^{\pi/2} \sin^n(x)\, dx = \frac{n-1}{n} \int_0^{\pi/2} \sin^{n-2}(x)\, dx.$$

(b) Find the length of the curve $f(x) = \displaystyle\int_0^x \sqrt{\left(\sin^2(t)+2\right)^2 - 1}\, dt$ for $0 \leq x \leq \frac{\pi}{4}$.

4. (12 pts) Consider the improper integral $\int_0^1 x^p \ln(x)\, dx$, where p is a constant.

(a) The integral converges for $p > -1$. For $p > -1$, determine the value the integral approaches in terms of p. (Justify your work).

(b) For $p = -1$, does the integral converge or diverge?
If it converges, give it's value. If it diverges, explain why.

5. (14 points) You run out of water balloons. So you devise a scheme to dump a large bucket of water on your instructor's head instead. Here is your plan:

   (a) A tank full of rainwater is outside your dorm. The shape of the water in the tank is described as follows:

   Consider the region R in the first quadrant of the xy-plane bounded by $y = x^2$, $y = 1$ and the y-axis (lengths are in meters). The water in the full tank is in the shape of the solid obtained by rotating R about the y-axis.

   You plan to pump all the water to the top of the tank and over the edge into your bucket (the bucket is large enough to hold all the water you pump out).

   (b) Once all the water is in your bucket. The full bucket is lifted by cable to the roof of your dorm, where you will wait for your instructor to walk by. The cable weighs 5 Newtons per meter and the *empty* bucket weighs 100 Newtons. The top of the building is 20 meters high.

   Recall the density of water is 1000 kg/m^3 and gravity is 9.8 m/s^2.

   Find the total amount of work done altogether in pumping out the water and then lifting the full bucket to the roof of your dorm. (Give your final answer as a decimal in Joules).