6.4 and 6.5 Review

My reviews and review sheets are not meant to be your only form of studying. It is vital to your success on the exams that you carefully go through and understand ALL the homework problems, worksheets and lecture material. Hopefully this review sheet will remind you of some of the key ideas of these sections.

1. Work
   - Understand the basics of how to use integrals to study work.
   - If x represents a distance and $f(x)$ represents the force on an object at a given distance, then
     $$\text{“the work required to move the object from } x = a \text{ to } x = b\text{”} = \int_a^b f(x)\ dx$$
     That is,
     $$\text{Work} = \int_a^b \text{Force}\ dx$$
   - We also used Newton's Second Law $F = ma$. Pay attention to units (I discuss the differences in class).
   - We looked at three major work examples. The first, springs, fit the above description nicely. The other two required a little more work to derive.
     (a) Springs
       i. x = distance beyond natural length.
       ii. Force $= f(x) = kx$, where k is the spring constant (you must find).
       iii. "Work to strecth from $x = a$ to $x = b$ beyond natural length" $= \int_a^b kx\ dx$
     (b) Lifting Cables
       i. x = the distance from the top of the building (or cliff).
       ii. k = the pounds per foot for the cable (or Newtons per meter).
       iii. Work to lift a slice $\approx k\Delta x$(distance lifted)
       iv. "Work to lift a rope of length $x = b$ all the way to the top" $= \int_0^b kx\ dx$
       v. Note: Here force is represented by kdx as we discussed in class.
     (c) Pumping Liquid
       i. x = the distance below the pump's spout = "distance from the top".
       ii. Work to lift pump out a slice $\approx$(weight of a slice)(distance lifted)
       = (weight per volume)(volume)(x)
       = (density $\times$ gravity)(area of fact of a slice $\times$ Δx)(x)
       iii. Work = (density)(gravity)$\int_a^b$(Area of a Horiz. Slice)$x\ dx$ ($x = a$ is the distance from the spout to the top of the liquid and $x = b$ is the distance from the spout to the lowest part that this being pumped out).
       iv. density for water = 1000 kg/m^3.
       v. gravity on Earth = 9.8 m/s^2.
       vi. For standard units, (density)(gravity) = 62.5 lbs/ft^3.
       vii. It is often useful to draw the 'trough' or 'container' on its side. Then find the equations for the curves. This will help you find a formula for the area of a horizontal slice.
     (d) NOTE: For all of these, it is good to have an understanding of how we use integral calculus. That is, we break up a problem into to small 'slices' and we approximate each slice. We approximate in such a way that more slices lead to better approximations. Finally, with the formula for each slice, we let the number of slices go to infinity. In this manner, we can represent the exact answer to a problem in terms of integrals.

2. Average Value

- Understand how to compute the average value of function on a given integral and how this value relates to areas and the mean value theorem.
- If $f(x)$ is a function, then we define the average value of $f(x)$ from $x = a$ to $x = b$ by

$$f\text{ave} = \frac{1}{b-a}\int_a^b f(x)\ dx$$

- We can interpret $f\text{ave}$ as the height we should use to get a perfect approximation of the area under the curve with only one rectangle.
- If the function is continuous from $x = a$ to $x = b$, then it must cross the horizontal line which represents the average values. This is the Mean Value Theorem for Integrals which is stated more techniquely as: If $f(x)$ is continuous from $x = a$ to $x = b$, then there is a value c between $x = a$ and $x = b$ where $f(c) = \frac{1}{b-a}\int_a^b f(x)\ dx$