Integrating Powers of Trig

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + C$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + C$$

Even powers (half-angle identity):

$$\int \cos^2(x)\,dx = \frac{1}{2}\int 1 + \cos(2x)\,dx = \frac{1}{2}x + \frac{1}{4}\sin(2x) + C$$

$$\int \sin^2(x)\,dx = \frac{1}{2}\int 1 - \cos(2x)\,dx = \frac{1}{2}x - \frac{1}{4}\sin(2x) + C$$

$$\begin{aligned}
\int \cos^4(x)\,dx &= \int [\frac{1}{2}(1+\cos(2x))]^2\,dx = \frac{1}{4}\int 1 + 2\cos(2x) + \cos^2(2x)\,dx \\
&= \frac{1}{4}x + \frac{1}{4}\sin(2x) + \frac{1}{4}\int \cos^2(2x)\,dx \\
&= \frac{1}{4}x + \frac{1}{4}\sin(2x) + \frac{1}{8}\int 1 + \cos(4x)\,dx \\
&= \frac{1}{4}x + \frac{1}{4}\sin(2x) + \frac{1}{8}x + \frac{1}{32}\sin(4x) + C \\
&= \frac{3}{8}x + \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x) + C
\end{aligned}$$

You can do $\sin^4(x)$ and $\sin^2(x)\cos^2(x)$ is a similar way as above.

Odd powers (identity then substitution):

$$\int \cos^3(x)\,dx = \int \cos^2(x)\cos(x)\,dx = \int (1 - \sin^2(x))\cos(x)\,dx$$

then use $u = \sin(x)$ to get

$$\int 1 - u^2 du = u - \frac{1}{3}u^3 + C$$

so

$$\int \cos^3(x)\,dx = \sin(x) - \frac{1}{3}\sin^3(x) + C$$

Here is another example

$$\int \cos^2(x)\sin^3(x)\,dx = \int \cos^2(x)\sin^2(x)\sin(x)\,dx = \int \cos^2(x)(1-\cos^2(x))\sin(x)\,dx$$

then use $u = \cos(x)$ to get

$$\int -u^2(1-u^2)du = -\frac{1}{3}u^3 + \frac{1}{5}u^5 + C$$

so

$$\int \cos^2(x)\sin^3(x)\,dx = -\frac{1}{3}\cos^3(x) + \frac{1}{5}\cos^5(x) + C$$