SOL'NS

Basic Integration Quiz Sheet

The following pages of integrals all can be evaluated by either simplification or u-substitution. The first 2 pages contain indefinite integrals. The last page contains definite integrals. By the end of the third week of class you should be able to complete the first 2 pages in 10-15 minutes and the last page in 10-15 minutes. So you should be able to complete these types of integral problems in about 1 minute or less each.

Note that our current methods are limited to these types of problems, there are lots of integrals we still are unable to do. (This means, on the first exam I can only ask you to evaluate integrals that can be completed using simplification or u-substitution.)

Evaluate all the following:

1. $\displaystyle\int 3x^{10} - \frac{\sqrt{x}}{x^2} + 4\, dx$

$= \int 3x^{10} - \frac{x^{1/2}}{x^2} + 4\, dx$

$= \int 3x^{10} - x^{-3/2} + 4\, dx$

$= \frac{3}{11}x^{11} - \frac{1}{-\frac{3}{2}+1}x^{-\frac{3}{2}+1} + 4x + C$

$= \boxed{\frac{3}{11}x^{11} + \frac{2}{\sqrt{x}} + 4x + C}$

2. $\displaystyle\int dx$

$= \int 1\, dx$

$= \boxed{x + C}$

3. $\displaystyle\int \sin(\tan(x))\sec^2(x)\, dx$

$u = \tan(x)$

$du = \sec^2(x)\, dx$

$dx = \frac{du}{\sec^2(x)}$

$= \int \sin(u)\, du$

$= -\cos(u) + C$

$= \boxed{-\cos(\tan(x)) + C}$

4. $\displaystyle\int x^7(1+x^8)^{31}\, dx$

$u = 1 + x^8$

$du = 8x^7 dx$

$dx = \frac{du}{8x^7}$

$= \int \cancel{x^7}\, u^{31} \frac{du}{8\cancel{x^7}}$

$= \frac{1}{8}\int u^{31}\, du$

$= \frac{1}{8}\frac{1}{32}u^{32} + C$

$= \boxed{\frac{1}{256}(1+x^8)^{32} + C}$

5. $\displaystyle\int \tan(x) + \frac{\sin(x)}{\cos^2(x)} + 13xe^{x^2}\, dx$

$\int \tan(x)dx = \ln|\sec(x)| + C_1$

$\int \frac{\sin(x)}{\cos^2(x)}dx = \int \tan(x)\sec(x)dx = \sec(x) + C_2$

$u = x^2$

$du = 2x\, dx$

$dx = \frac{du}{2x}$

$\int 13xe^{x^2}dx = \int 13x e^{u}\frac{du}{2x}$

$= \frac{13}{2}\int e^u du = \frac{13}{2}e^u + C_3$

$= \frac{13}{2}e^{x^2} + C_3$

$\boxed{\text{ANS} = \ln|\sec(x)| + \sec(x) + \frac{13}{2}e^{x^2} + C}$

6. $\displaystyle\int (5x^4 - 6x)\sqrt{x^5 - 3x^2 + 1}\, dx$

$u = x^5 - 3x^2 + 1$

$du = 5x^4 - 6x\, dx$

$dx = \frac{du}{5x^4 - 6x}$

$= \int \sqrt{u}\, du$

$= \int u^{1/2} du$

$= \frac{1}{3/2}u^{3/2} + C$

$= \boxed{\frac{2}{3}(x^5 - 3x^2 + 1)^{3/2} + C}$

7. $\int x(1+x)^5\,dx$

$u = 1+x$
$du = dx$
$dx = du$
Note: $x = u-1$

$= \int (u-1)\,u^5\,du$

$= \int u^6 - u^5\,du$

$= \frac{1}{7}u^7 - \frac{1}{6}u^6 + C$

$= \boxed{\frac{1}{7}(1+x)^7 - \frac{1}{6}(1+x)^6 + C}$

8. $\int \frac{\sqrt{3x^3} + 4x^2 - 11x}{\sqrt{x}}\,dx$

$= \int \frac{\sqrt{3}\,x^{3/2}}{x^{1/2}} + \frac{4x^2}{x^{1/2}} - \frac{11x}{x^{1/2}}\,dx$

$= \int \sqrt{3}\,x + 4x^{3/2} - 11x^{1/2}\,dx$

$= \frac{\sqrt{3}}{2}x^2 + \frac{4}{5/2}x^{5/2} - \frac{11}{3/2}x^{3/2} + C$

$= \boxed{\frac{\sqrt{3}}{2}x^2 + \frac{8}{5}x^{5/2} - \frac{22}{3}x^{3/2} + C}$

9. $\int \frac{x^5}{\sqrt{1+x^3}}\,dx$

$u = 1+x^3$
$du = 3x^2dx$
$dx = \frac{du}{3x^2}$
Note: $x^3 = u-1$

$= \int \frac{\cancel{x^5}\,x^3}{\sqrt{u}}\,\frac{du}{3\cancel{x^2}}$

$= \frac{1}{3}\int \frac{u-1}{\sqrt{u}}\,du$

$= \frac{1}{3}\int \frac{u}{u^{1/2}} - \frac{1}{u^{1/2}}\,du = \frac{1}{3}\int u^{1/2} - u^{-1/2}\,du$

$= \frac{1}{3}\left(\frac{1}{3/2}u^{3/2} - \frac{1}{1/2}u^{1/2} + C_1\right)$

$= \boxed{\frac{2}{9}(1+x^3)^{3/2} - \frac{2}{3}(1+x^3)^{1/2} + C}$

10. $\int \cos(x)\sin(x)\,dx$

$u = \sin(x)$
$du = \cos(x)dx$
$dx = \frac{du}{\cos(x)}$

$= \int u\,du$

$= \frac{1}{2}u^2 + C$

$= \boxed{\frac{1}{2}\sin^2(x) + C}$

11. $\int \sin(13x)\,dx$

$u = 13x$
$du = 13dx$
$dx = \frac{1}{13}du$

$= \int \sin(u)\,\frac{du}{13}$

$= \frac{1}{13}\int \sin(u)\,du$

$= -\frac{1}{13}\cos(u) + C = \boxed{-\frac{1}{13}\cos(13x) + C}$

12. $\int e^{7x}\,dx$

$u = 7x$
$du = 7dx$
$dx = \frac{du}{7}$

$= \int e^u\,\frac{du}{7}$

$= \frac{1}{7}e^u + C$

$= \boxed{\frac{1}{7}e^{7x} + C}$

13. $\int \cos(\frac{1}{4}x)\,dx$

$u = \frac{1}{4}x$
$du = \frac{1}{4}dx$
$dx = 4du$

$= \int \cos(u)\,4\,du$

$= 4\sin(u) + C$

$= \boxed{4\sin(\frac{1}{4}x) + C}$

14. $\int \sin(-5x)\,dx$

$u = -5x$
$du = -5dx$
$dx = \frac{du}{-5}$

$= \int \sin(u)\,\frac{du}{-5}$

$= -\frac{1}{5}(-\cos(u)) + C$

$= \boxed{\frac{1}{5}\cos(-5x) + C}$

15. $\int e^{101x}\,dx$

$u = 101x$
$du = 101dx$
$dx = \frac{du}{101}$

$= \int e^u\,\frac{du}{101}$

$= \frac{1}{101}e^u + C$

$= \boxed{\frac{1}{101}e^{101x} + C}$

16. $\int \cos(2x)\,dx$

$u = 2x$
$du = 2dx$
$dx = \frac{du}{2}$

$= \int \cos(u)\,\frac{du}{2}$

$= \frac{1}{2}\sin(u) + C$

$= \boxed{\frac{1}{2}\sin(2x) + C}$

17. $\int_0^{(\frac{\pi}{2})^{1/3}} x^2 \sin(x^3)\, dx$

$u = x^3$

$du = 3x^2 dx$

$dx = \frac{du}{3x^2}$

$x = 0 \quad u = 0$

$x = (\frac{\pi}{2})^{1/3} \quad u = \pi/2$

$\int_0^{\pi/2} \sin(u) \frac{du}{3}$

$\frac{1}{3} \int_0^{\pi/2} \sin(u) du$

$= -\frac{1}{3} \cos(u) \Big|_0^{\pi/2}$

$= (-\frac{1}{3}\cos(\pi/2)) - (-\frac{1}{3}\cos(0)) = (0) - (-1/3(1))$

$\boxed{\frac{1}{3}}$

18. $\int_0^{\frac{\pi}{2}} e^{-3\cos(x)} \sin(x)\, dx$

$u = -3\cos(x)$

$du = 3\sin(x) dx$

$dx = \frac{du}{3\sin(u)}$

$x = 0 \Rightarrow u = -3$

$x = \pi/2 \Rightarrow u = 0$

$= \int_{-3}^{0} e^u \frac{du}{3}$

$= \frac{1}{3} \int_{-3}^{0} e^u du$

$= \frac{1}{3} e^u \Big|_{-3}^{0}$

$= (\frac{1}{3} e^0) - (\frac{1}{3} e^{-3})$

$= \boxed{\frac{1}{3} - \frac{1}{3} e^{-3}} \approx \boxed{0.316737643877}$

19. $\int_{-10}^{-3} e^{\frac{1}{10}x}\, dx$

$u = \frac{1}{10} x$

$du = \frac{1}{10} dx$

$dx = 10\, du$

$x = -10 \Rightarrow u = -1$

$x = -3 \Rightarrow u = -\frac{3}{10}$

$= \int_{-1}^{-3/10} e^u\, 10 du$

$= 10 \int_{-1}^{-3/10} e^u du$

$= 10 e^u \Big|_{-1}^{-3/10} = \boxed{(10 e^{-3/10}) - (10 e^{-1})}$

$\approx \boxed{3.7293877951}$

20. $\int_{\pi/3}^{\pi/4} \sin(-\frac{7}{8}x)\, dx$

$u = -\frac{7}{8} x$

$du = -\frac{7}{8} dx$

$dx = -\frac{8}{7} du$

$x = \pi/3 \Rightarrow u = -\frac{7\pi}{24}$

$x = \pi/4 \Rightarrow u = -\frac{7\pi}{32}$

$= \int_{-\frac{7\pi}{24}}^{-\frac{7\pi}{32}} \sin(u) \cdot -\frac{8}{7} du$

$= -\frac{8}{7} \int_{-\frac{7\pi}{24}}^{-\frac{7\pi}{32}} \sin(u) du$

$= \frac{8}{7} \cos(u) \Big|_{-\frac{7\pi}{24}}^{-7\pi/32}$

$= \boxed{\left(\frac{8}{7} \cos(-\frac{7\pi}{32})\right) - \left(\frac{8}{7} \cos(-\frac{7\pi}{24})\right)} \approx \boxed{0.1877132}$

21. $\int_2^3 \frac{x}{\sqrt{4+x^2}}\, dx$

$u = 4 + x^2$

$du = 2x\, dx$

$dx = \frac{du}{2x}$

$x = 2 \Rightarrow u = 8$

$x = 3 \Rightarrow u = 13$

$= \int_8^{13} \frac{\cancel{x}}{\sqrt{u}} \frac{du}{2\cancel{x}}$

$= \frac{1}{2} \int_8^{13} u^{-1/2} du$

$= \frac{1}{2} \frac{1}{1/2} u^{1/2} \Big|_8^{13} = u^{1/2} \Big|_8^{13}$

$= \boxed{13^{1/2} - 8^{1/2}} \approx \boxed{0.777124150718}$

22. $\int_1^5 \frac{x^3 - 2x^2 + x^{5/2}}{x^{1/3}}\, dx$

$= \int_1^5 \frac{x^3}{x^{1/3}} - \frac{2x^2}{x^{1/3}} + \frac{x^{5/2}}{x^{1/3}} dx$

$= \int_1^5 x^{8/3} - 2x^{5/3} + x^{13/6} dx$

$= \frac{3}{11} x^{11/3} - 2\frac{3}{8} x^{8/3} + \frac{6}{19} x^{19/6} \Big|_1^5$

$\approx \boxed{96.6367506351}$

23. $\int_2^{2e} \frac{3-x}{2x}\, dx$

$= \int_2^{2e} \frac{3}{2x} - \frac{x}{2x} dx$

$= \frac{3}{2} \int_2^{2e} \frac{1}{x} dx - \int_2^{2e} \frac{1}{2} dx$

$= \frac{3}{2} \ln(x) \Big|_2^{2e} - \frac{1}{2} x \Big|_2^{2e}$

$= (\frac{3}{2} \ln(2e) - \frac{3}{2} \ln(2)) - (\frac{1}{2} 2e - \frac{1}{2} 2)$

$= \boxed{5/2 - e} \approx \boxed{-0.21828182845 9}$

24. $\int_1^e \frac{(\ln(x))^3}{x}\, dx$

$u = \ln(x)$

$du = \frac{1}{x} dx$

$dx = x\, du$

$x = 1 \Rightarrow u = 0$

$x = e \Rightarrow u = 1$

$= \int_0^1 u^3 du$

$= \frac{1}{4} u^4 \Big|_0^1$

$= (\frac{1}{4}(1)^4) - (\frac{1}{4}(0^4))$

$= \boxed{\frac{1}{4}}$