I have already posted a great deal of review material for exam 2 with all the basic facts, formulas and concepts. As students have come to me with questions on 'harder'/'obscure' problems from deep in the exam archives, some common themes have popped up. This posting is just to expand (revisit) some of these concepts in answer to common student questions of the last week. This review sheet is just concepts (see elsewhere for the formulas).

13.3 and 13.4: We discussed many ways to measure a curve in 3D including the unit tangent, the unit normal, the binormal, osculating plane, normal plane, acceleration, velocity, speed, normal component of acceleration, and tangential component of acceleration. From what we discussed in class, what you saw in homework, and from my other review sheets, you should be able to find/compute what is implied by these terms. Based on a couple of student questions this week, let me remind you of several conceptual facts:

- $\mathbf{T}(t)$ is the unit tangent and $\mathbf{N}(t)$ is the unit normal (it points 'inwardly').
- The plane that 'contains' both $\mathbf{T}(t)$ and $\mathbf{N}(t)$ at a point on a curve is called the osculating plane. (Osculate means 'kissing') The osculating plane best approximates the plane on which the motion is occurring at that particular time, because it contains the tangent vector and the inwardly pointing normal vector.
- $\mathbf{r}'(t)$ and $\mathbf{T}(t)$ point in the same direction. So $\mathbf{r}'(t)$ is also parallel to the osculating plane.
- $\mathbf{r}''(t)$ is the acceleration vector. We learned that it can be written as a combination of $\mathbf{T}$ and $\mathbf{N}$ (Namely, we learned $\mathbf{r}'' = a_T\mathbf{T} + a_N\mathbf{N}$). So $\mathbf{r}''(t)$ is also parallel to the osculating plane.
  A couple of special cases worth noting:
  - If $a_T = 0$, then the acceleration vector is parallel to $\mathbf{N}(t)$.
  - If $a_N = 0$, then the acceleration vector is parallel to $\mathbf{T}(t)$.
- Collecting facts from above: $\mathbf{T}(t)$, $\mathbf{N}(t)$, $\mathbf{r}'(t)$, and $\mathbf{r}''(t)$ are all parallel to the osculating plane!
- The binormal, $\mathbf{B}(t)$, is orthogonal to the osculating plane. Thus, $\mathbf{B}(t)$ is orthogonal to all of the vectors $\mathbf{T}(t)$, $\mathbf{N}(t)$, $\mathbf{r}'(t)$, and $\mathbf{r}''(t)$.
- A few old exams have problems where the motion is all contained on the same plane. For example, $\mathbf{r}(t) = \langle 5t^2, 0, t^3\rangle$ would be a curve that is always on the xz-plane. Thus, $\mathbf{r}'(t)$, $\mathbf{r}''(t)$, $\mathbf{T}(t)$ and $\mathbf{N}(t)$ are all parallel to the xz-plane. And so the binormal is orthogonal to the xz-plane (so it must be parallel to the y-axis).

14.1, 14.3, 14.4, 14.7: We discussed surfaces, partial derivatives, tangent planes (also linear approximations and differentials), local max/min, and global max/min. My review sheets cover all of these topics in detail with examples. There isn't much more to say, but I will recap the skills I have been asked about most often in the last week of review:

- Be good at doing partial derivatives (be able to handle all situations from class and homework: product rule, quotient rule, chain rule, and implicit).
- Know how to find tangent planes.
- Be able to find critical points.
  You need to find points (a, b) where BOTH $f_x(a, b) = 0$ AND $f_y(a, b) = 0$. Understand the logic of these situation.
  1. Simplify $f_x(x, y) = 0$ into something of the form $y =??$ or $x =??$. (Also try factoring).
  2. For each equation/solution that made $f_x(x, y) = 0$, substitute into $f_y(x, y) = 0$ and solve.
  3. After you've tried all cases, check to make sure that all your points (a, b) make both partial derivatives zero.
- Understand how to use the second derivative test.
- For global max/min questions, you are trying to find the possible **candidates** that could be global max/min values. Once you have limited the list to only a few possible points, you plug each of them into $z = f(x, y)$ and see which give the biggest and which give the smallest z values.
  1. Find all critical points inside the region. Those are the only points from the inside that could correspond to global max/min values.
  2. For each boundary:
     (a) Find an equation involving x and y that describes the boundary (typically $y = f(x)$ or $x = g(y)$)
     (b) Substitute this equation into $z = f(x, y)$, which will result in an equation that has only two variables (z and x or z and y).
     (c) Find z' and set this equal to zero. This will give the critical numbers over this boundary.
     (d) The endpoints of this boundary and the critical numbers you just found are the only points from this boundary that could correspond to global max/min values.
- For applied problems:
  1. Label everything, draw a picture.
  2. Identify the **objective**: What are you being asked to optimize? (Does it say find the minimum distance, volume, cost, ...?). Write an expression that is equal to the objective (this might involve 3 variables).
  3. Identify any **constraints**: What else are you given? Write down these facts in an equation.
  4. Solve for one variable in terms of the others in the constraint, and substitute this fact into the objective.
  5. That will give you a two variable function to optimize. Use the techniques already discussed to find max/min (start with the critical points)

15.1-15.5: We discussed double integrals over general regions and polar regions with applications to volume, area, average value, and center of mass. Here are the most common questions from review over the last week:

- Understand how to graph lines: $y = mx + b$, basic powers and roots: $y = 3x^2$, $y = 4x^3$, $y = 5\sqrt{x}$, standard functions from precalculus: $\ln(x)$, e^x, $\sin(x)$, $\cos(x)$. You need these skills so that you can graph regions (essential for setting up double integrals).
- Understand the concepts of top/bottom regions ($g_1(x) \leq y \leq g_2(x)$) and left/right regions ($h_1(y) \leq x \leq h_2(y)$).
- Be able to reverse the order of integration.
- Understand how to work with polar regions.
  1. Use $x = r\cos(\theta)$, $y = r\sin(\theta)$, $x^2 + y^2 = r^2$, $dA = r dr d\theta$.
  2. Describe the region (find intersections to get bounds for θ).
  3. Think 'inside/outside' for bounds on r.
- Know application formulas for volume, area, average value, and center of mass.
- If a problem is given in words:
  1. Solve for $z =$??? (wherever a z occurs). This is your integrand. If there is more than one z, then you are going to have to do more than one double integral.
  2. Graph the given x-y equations in the xy-plane. This is your region of integration.
  3. If the given x-y equations don't form a region, then reread the problem. If the solid is bounded between two surfaces, then you will need to find their intersection (where the $z =$?? equations are equal). This will give you your x-y region of integration.
  4. Once you have the region and the integrand, you are all set. Make the bounds and integrate.