MATH 126

Exam II Review - Hints and Answers

1. (a) $a_T = \dfrac{2}{\sqrt{6}}$

   (b) $t = \pm\dfrac{1}{\sqrt[6]{2}}$

2. $a_T = \dfrac{4t}{\sqrt{4t^2+26}}, a_N = \dfrac{\sqrt{104}}{\sqrt{4t^2+26}}$

3. (a) the domain is the half plane above (not including) the line $y = -x$

   (b) $f_{xy}(x, y) = e^y + \dfrac{1}{(x+y)^2}$

4. (a) $f_y(x, y) = x^2 + x\cos y + \dfrac{2y}{x-y^2}$

   (b) $f_{xy}(x, y) = 2x + \cos y - \dfrac{2y}{(x-y^2)^2}$

5. $f_x = 4x^3y^3 - 3y^2 + 20x^4 + (e^{x^3-x})(3x^2-1)\ln y,$

   $f_{xx} = 12x^2y^3 + 80x^3 + (\ln y)e^{x^3-x}((3x^2-1)^2 + 6x),$

   $f_{xy} = 12x^3y^2 - 6y + e^{x^3-x}(3x^2-1)y^{-1}$

6. (a) The level curve consists of the two lines $y = \pm\sqrt{\dfrac{2}{3}}x.$

   (b) $z = \dfrac{4}{3}(x-2) - (y-1) + 3$

   (c) $f(1.9, 1.2) \approx 2.66666.....$

7. (a) The domain is the half-plane below the line $y = 2x$, excluding the line $y = 2x - 1$.

   (b) $-2e(x-e) + 5e(y-e) + 3e^2 = z$

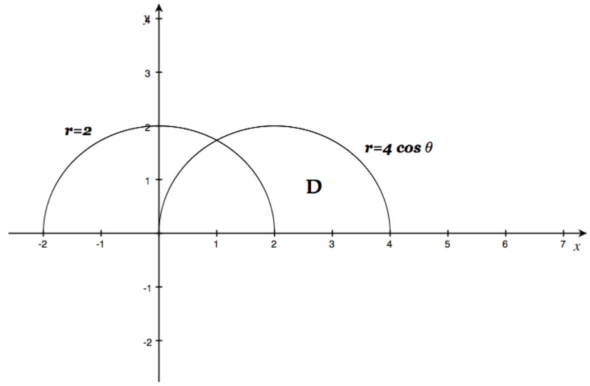
   (c) $f(3, 3) \approx 9e \approx 24.464536....$

8. The point (-2,3) gives a saddle point, and the point (2,3) gives a local minimum.

9. The glass should have horizontal length 8.0505 meters and vertical length 4.0252 meters. The other dimension of the pool should be 30.85989 meters.

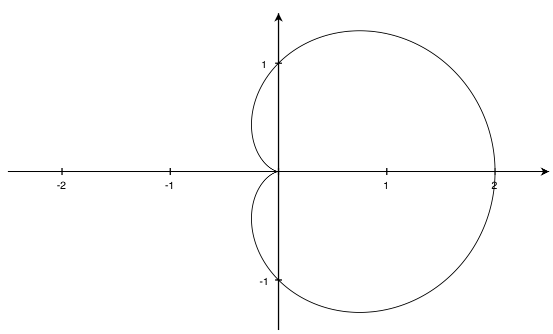
10. The point (0,0) gives a saddle point, and the point (1,1) gives a local maximum.

11. (a) Here's what D looks like:



(b) $A(D) = \iint\limits_D 1\, dA = \int_0^{\pi/3} \int_2^{4\cos\theta} r\, dr\, d\theta = ... = \frac{2\pi}{3} + \sqrt{3}$

12. (a) Here's what the cardiod looks like:



The area of the region bounded by the x-axis and $r = 1 + \cos\theta$ from $\theta = 0$ to $\theta = \pi$ is:

$$A = \int_0^{\pi} \int_0^{1+\cos\theta} r\, dr\, d\theta = ... = \frac{3\pi}{4}.$$

(b) $V = \iint\limits_R y\, dA = \int_0^{\pi/2} \int_1^{1+\cos\theta} r^2 \sin\theta\, dr\, d\theta = ... = \frac{11}{12}$

13. HINT: Change the order of integration:

$$\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y}\, dy\, dx = \int_0^4 \int_0^{\sqrt{4-y}} \frac{xe^{2y}}{4-y}\, dx\, dy = ... = \frac{e^8 - 1}{4}$$

14. The mass of the lamina will be a function of a:

$$m(a) = \int_1^2 \int_{ax}^{2ax} \frac{1}{x} + \frac{1}{y^2}\, dy\, dx = ... = a + \frac{\ln 2}{2a}.$$

Now, find the value of a that minimizes this function: compute $m'(a)$, set it equal to 0, and solve for a to find the critical numbers of $m(a)$. Then use the second derivative test (for a **single-variable** function) to show that the critical number you get gives a minimum.

ANSWER: $a = \sqrt{\dfrac{\ln 2}{2}}$

15. $m = \displaystyle\int_0^1 \int_0^{1-x} 3e^{-2x-3y}\, dy\, dx = ... = \frac{1}{e^3} + \frac{1}{2} - \frac{3}{2e^2}$

16. $I = 8\ln 8 - 16 + e$

17. $V = \displaystyle\iint_R e^{-(x^2+y^2)}\, dA = \int_0^{\pi} \int_1^2 e^{-r^2} r\, dr\, d\theta = ... = \frac{\pi}{2}(e^{-1} - e^{-4})$