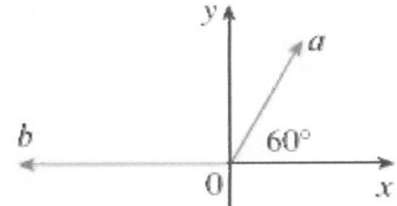
1. (11 points)

(a) The forces **a** and **b** are the pictured. If $|\mathbf{a}| = 80$ N and $|\mathbf{b}| = 100$ N, find the angle the **resultant** force makes with the positive x-axis.
(Give your answer rounded to the nearest degree).



$\vec{a} = \langle 80\cos(60°), 80\sin(60°)\rangle = \langle 40, 40\sqrt{3}\rangle$

$\vec{b} = \langle -100, 0\rangle$

RESULTANT FORCE $= \vec{a} + \vec{b}$

$= \langle -60, 40\sqrt{3}\rangle = \vec{u}$

ANGLE WITH POSITIVE X-AXIS? CAN USE $\vec{v} = \langle 1, 0\rangle$ AND $\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos\theta$

$\Rightarrow -60 = \sqrt{60^2 + 40^2\cdot 3}\,\sqrt{1}\,\cos\theta$

$-60 = \sqrt{8400}\,\cos\theta$

$\cos(\theta) = \frac{-60}{\sqrt{8400}} = \frac{-60}{20\sqrt{21}} = -\frac{3}{\sqrt{21}} = -\sqrt{\frac{3}{7}} = -\frac{\sqrt{21}}{7}$

$\theta = \cos^{-1}\left(\frac{-60}{\sqrt{8400}}\right) \approx 130.8933946 \approx \boxed{131°}$

(2.2845 radians)

(b) Find the center and radius of the sphere with points $P(x, y, z)$ such that the distance from P to $A(0, 0, 2)$ is triple the distance from P to $B(0, 0, 0)$.

$\sqrt{x^2 + y^2 + (z-2)^2} = 3\sqrt{x^2 + y^2 + z^2}$

$x^2 + y^2 + z^2 - 4z + 4 = 9x^2 + 9y^2 + 9z^2$

$4 = 8x^2 + 8y^2 + 8z^2 + 4z$

$\frac{1}{16} + \frac{1}{2} = x^2 + y^2 + z^2 + \frac{1}{2}z + \frac{1}{16}$

$\frac{9}{16} = x^2 + y^2 + (z + \frac{1}{4})^2$

$\boxed{\text{CENTER} = (0, 0, -\tfrac{1}{4}) \qquad \text{RADIUS} = \tfrac{3}{4}}$

2. (12 pts)

(a) Find the equation for the plane that contains the line $x = t$, $y = 1 - 2t$, $z = 4$ and the point [redacted]. $(3,-1,5)$

$t=0$ $\quad$ $t=1$

3 POINTS: $P(3,-1,5)$, $Q(0,1,4)$, $R(1,-1,4)$

2 VECTORS PARALLEL TO DESIRED PLANE: $\vec{PQ} = \langle -3, 2, -1 \rangle$

$\vec{QR} = \langle 1, -2, 0 \rangle$ ← DIRECTION VECTOR FROM LINE

NORMAL: $\vec{PQ} \times \vec{QR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & 2 & -1 \\ 1 & -2 & 0 \end{vmatrix} = (0-2)\vec{i} - (0-1)\vec{j} + (6-2)\vec{k}$

$= \langle -2, -1, 4 \rangle$

PLANE:

$$\boxed{\begin{aligned} &-2(x-3) - (y+1) + 4(z-5) = 0 \\ &-2x + 6 - y - 1 + 4z - 20 = 0 \\ &-2x - y + 4z = 15 \end{aligned}}$$

← ANY NONZERO CONSTANT MULTIPLE CORRECT WITH ANY POINT FROM THE PLANE IN PLACE OF $(3,-1,5)$.

ALL SOL'NS EXPAND AND SIMPLIFY TO THIS.

(b) Consider the line L_1 that goes through the points $(-3, 3, 0)$ and $(-1, 4, 6)$ and the line L_2 that is given by $x = 2 + t$, $y = 3 - 2t$, $z = 19 + 7t$. These lines are not parallel.
Are L_1 and L_2 intersecting or skew? Justify your answer by either finding the point of intersection or showing that there is no intersection point.

L_1: POINT $(-3,3,0)$ DIRECTION: $\langle 2,1,6 \rangle$

USE DIFFERENT PARAMETER!!

$x = -3 + 2u$, $y = 3 + u$, $z = 6u$

INTERSECT? (i) $2 + t = -3 + 2u \Rightarrow t = -5 + 2u$

(ii) $3 - 2t = 3 + u \Rightarrow -2t = u$

(iii) $19 + 7t = 6u$

(i) & (ii) $\Rightarrow -2(-5+2u) = u \Rightarrow 10 - 4u = u \Rightarrow 10 = 5u \Rightarrow u = 2$

$-2t = u \Rightarrow t = -1$

CHECK (iii) $19 + 7(-1) = 12$ $\quad$ $6(2) = 12$ ✓

YES, THEY INTERSECT AT

$(1, 5, 12)$.

3. (14 pts)

(a) Find a vector $\mathbf{v}$ such that
1. $\mathbf{v}$ is parallel to the tangent line to $x = 6\ln(t-4)$, $y = t^2 - 3t$ at the point $(0, 10)$, and
2. $|\mathbf{v}| = 5$.

TANGENT VECTOR: $\langle x'(t), y'(t) \rangle = \langle \frac{6}{t-4}, 2t-3 \rangle$

POINT: $(x,y) = (0,10) \Rightarrow 0 = 6\ln(t-4)$ & $10 = t^2 - 3t \Rightarrow$ $t=5$

$t-4=1$

TANGENT VECTOR $= \langle \frac{6}{5-4}, 2(5)-3 \rangle = \langle 6, 7 \rangle = \vec{u}$

OR FIND $\frac{dy}{dx} = \frac{7}{6}$ $\Downarrow$ $\langle 6,7 \rangle$

NOW WE WANT TO SCALE TO LENGTH 5.
WE FIRST SCALE TO LENGTH ONE THEN MULTIPLY BY 5, NAMELY $\frac{5}{|\vec{u}|}\vec{u}$.

$$\frac{5}{\sqrt{36+49}} \langle 6, 7 \rangle = \boxed{\left\langle \frac{30}{\sqrt{85}}, \frac{35}{\sqrt{85}} \right\rangle}$$

OR THE OPPOSITE DIRECTION $\left\langle \frac{-30}{\sqrt{85}}, \frac{-35}{\sqrt{85}} \right\rangle$

(b) The polar curve $r = 2 + \cos(3\theta)$ intersects the negative y-axis at only one point, P. Find the equation for the tangent line to the curve at this point P.
(Put your answer in the form $y = m(x - x_0) + y_0$).

r IS ALWAYS POSITIVE (BECAUSE $\cos(3\theta) \geq -1$ ALWAYS) AND WE ARE TOLD THERE IS ONLY ONE NEG. y-AXIS INTERSECTION.
THUS, THIS INTERSECTION HAS TO BE GIVEN BY $\theta = 3\pi/2$

OR ANY OTHER EQUIVALENT FACING ANGLE $(\ldots, -\pi/2, 3\pi/2, 7\pi/2, \ldots)$

$$r = 2 + \cos(3\theta) \Rightarrow r(3\pi/2) = 2 + \cos\left(\tfrac{9\pi}{2}\right) = 2 + 0 = 2$$

$$\frac{dr}{d\theta} = -3\sin(3\theta) \Rightarrow \left.\frac{dr}{d\theta}\right|_{3\pi/2} = -3\sin(9\pi/2) = -3$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \left.\frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}\right|_{\theta=3\pi/2} = \frac{(-3)(-1)+(2)(0)}{(-3)(0)-(2)(-1)} = \frac{3}{2}$$

$$r = 2,\ \theta = \tfrac{3\pi}{2} \Rightarrow x = 0,\ y = -2$$

TANGENT LINE: $\boxed{y = \frac{3}{2}(x-0) - 2 = \frac{3}{2}x - 2}$

4. (13 pts) Consider the vector function $\mathbf{r}(t) = \langle t\cos(3t), t^2, t\sin(3t)\rangle$.

(a) Describe the surface of motion for the resulting parametric curve.
(Eliminate the parameter and give the specific name of the surface of motion).

$x^2 + z^2 = t^2\cos^2(3t) + t^2\sin^2(3t) = t^2(\cos^2(3t) + \sin^2(3t)) = t^2$

So $x^2 + z^2 = y$ $\Rightarrow$ $\boxed{\text{CIRCULAR PARABOLOID}}$

(b) Find the parametric equations for the tangent line at $t = \pi$.

$\vec{r}(\pi) = \langle -\pi, \pi^2, 0\rangle$

$\vec{r}\,'(t) = \langle \cos(3t) - 3t\sin(3t),\ 2t,\ \sin(3t) + 3t\cos(3t)\rangle$

$\vec{r}\,'(\pi) = \langle -1, 2\pi, -3\pi\rangle$

$$\boxed{\begin{aligned} x &= -\pi - u \\ y &= \pi^2 + 2\pi u \\ z &= 0 - 3\pi u \end{aligned}}$$

(c) Find the curvature at $t = 0$.

$\vec{r}\,''(t) = \langle -3\sin(3t) - 3\sin(3t) - 9t\cos(3t),\ 2,\ 3\cos(3t) + 3\cos(3t) - 9t\sin(3t)\rangle$

$\vec{r}\,'(0) = \langle 1, 0, 0\rangle$

$\vec{r}\,''(0) = \langle 0, 2, 6\rangle$

$$\vec{r}\,'(0) \times \vec{r}\,''(0) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 0 \\ 0 & 2 & 6 \end{vmatrix} = (0-0)\vec{i} - (6-0)\vec{j} + (2-0)\vec{k} = \langle 0, -6, 2\rangle$$

$$\frac{|\vec{r}\,'(0) \times \vec{r}\,''(0)|}{|\vec{r}\,'(0)|^3} = \frac{\sqrt{0^2 + 36 + 4}}{(\sqrt{1})^3} = \boxed{\sqrt{40} = 2\sqrt{10}}$$